Math 4326/5322   **LINEAR ALGEBRA**   Spring 2019
Dr. Duval   **Homework**

Monday, February 11

Follow the separate general guidelines for Parts A,B,C. Be sure to include and label *all four* standard parts (a), (b), (c), (d) of Part A in what you hand in.

The Vector Space of Linear Maps
Section 3.A

A: Reading questions. Due by 2pm, Sun., 17 Feb.

1. Verify the following functions, described on pp. 52–53, are in fact linear maps: identity, differentiation, multiplication by x^2, backward shift.

2. Let's illustrate one part of result 3.5 (Linear maps and basis ...), namely the need for $v_1, \ldots, v_n$ to be a basis, with an example. First, explain why $(1,0,1), (0,1,1), (1,1,2)$ is **not** a basis of $\mathbf{F}^3$. Then show that there is **no** linear map $T: \mathbf{F}^3 \to \mathbf{F}^4$ such that $T((1,0,1)) = (2,0,1,9)$, $T((0,1,1)) = (1,9,6,6)$, $T((1,1,2)) = (3,9,6,12)$.

3. Verify that $S + T$ is a linear map from V to W whenever $S, T \in \mathcal{L}(V,W)$.

4. Verify the first distributive property on p. 56: $(S_1 + S_2)T = S_1T + S_2T$ whenever $T \in \mathcal{L}(U,V)$ and $S_1, S_2 \in \mathcal{L}(V,W)$.

B: Warmup exercises. For you to present in class. Due by the end of class Mon., 18 Feb.

Exercises 3.A: 1, 5, 8

Null Spaces and Ranges (part I):
Injectivity and Surjectivity
Section 3.B, pp. 59–62

A: Reading questions. Due by 2pm, Tue., 19 Feb.

1. Find the null space of the identity map, defined on p. 53. Is this map injective? Why or why not?

2. Find the range of the multiplication by x^2 map, defined on p. 53. Is this map surjective? Why or why not?

3. Identify, as precisely as you can, when we **use** the linearity of T in the proofs of results 3.14 (null space is subspace), 3.16 (injectivity is equivalent ...), and 3.19 (range is subspace). [Pinpoint the exact equations and statements that depend on linearity, and which part of the definition of linearity that is used in each case.]

4. Does surjectivity of a map $T \in \mathcal{L}(V,W)$ depend on V, W, both, or neither? If it does depend on V and/or W, give an example showing how changing V and/or W changes the surjectivity of T.

B: Warmup exercises. For you to present in class. Due by end of class Wed., 20 Feb.

Exercises 3.B: 1, 3.