Math 4326/5322 **LINEAR ALGEBRA** Spring 2019
Dr. Duval **Main Exercises 9**
due 2pm, Thursday, April 18

1. Let V be an inner product space, and let $T \in \mathcal{L}(V)$. Prove that if $\|Tv\| \geq \|v\|$ for every $v \in V$, then $5T - 2I$ is invertible.

2. Find vectors $w, z \in \mathbf{R}^2$ such that w is a scalar multiple of $(2, 4)$, z is orthogonal to $(2, 4)$, and $(1, 3) = w + z$.

3. Let n be a positive integer, and let $x_1, \ldots, x_n$ be real numbers. Prove that

$$(x_1 + \cdots + x_n)^2 \leq n(x_1^2 + \cdots + x_n^2).$$

4. Is the function that takes $\big((x_1, x_2), (y_1, y_2)\big) \in \mathbf{R}^2 \times \mathbf{R}^2$ to $x_1y_2 + x_2y_1$ an inner product on $\mathbf{R}^2$, or not? Prove your answer is correct.

5. (Graduate students only) Let V be an inner product space, and let $T \in \mathcal{L}(V)$ be injective. Define $\langle\langle \cdot, \cdot \rangle\rangle$ by

$$\langle\langle u, v \rangle\rangle = \langle Tu, Tv \rangle$$

for $u, v \in V$. Prove that $\langle\langle \cdot, \cdot \rangle\rangle$ is an inner product.