Math 4326/5322 | **LINEAR ALGEBRA** | Spring 2019

Dr. Duval

Main Exercises 6

due 2pm, Thursday, March 14

1. Assume A is an $m \times n$ matrix and C is an $n \times p$ matrix. Let j be an integer such that $1 \leq j \leq m$. Prove that

$$(AC)_{j,\cdot} = (A_{j,\cdot})C$$

in other words that the jth row of AC equals the jth row of A, times C.

2. Assume a is a $1 \times n$ matrix $(a_1 \ \cdots \ a_n)$, and assume C is an $n \times p$ matrix. Prove that

$$aC = a_1 C_{1,\cdot} + \cdots a_n C_{n,\cdot}$$

in other words that aC is a linear combination of the rows of C, with the coefficients from the linear combination given by a.

3. Assume V is finite-dimensional, and $S, T, U \in \mathcal{L}(V)$ and also assume that $STU = I$. Prove that S, T, and U are invertible. Then prove that $T^{-1} = US$.

4. Recall that when A is an $m \times n$ matrix and $b \in \mathbf{F}^m$, then $Ax = b$ corresponds to a system of linear equations in m equations in n unknowns. In the special case when $m = n$, prove that

$$0 \text{ is the only solution to the system } Ax = 0$$

if and only if

$$\text{the system } Ax = b \text{ has a solution for every choice of } b \in \mathbf{F}^n.$$

5. (Graduate students only) Assume V is finite-dimensional, and $T \in \mathcal{L}(V)$. Prove that

$$ST = TS \text{ for all } S \in \mathcal{L}(V)$$

if and only if $T = aI$ for some scalar I.