Math 4326/5322 | **LINEAR ALGEBRA** | Spring 2019

Dr. Duval | **Main Exercises 4**

due 2pm, Thursday, February 28

1. Suppose $T \in \mathcal{L}(V, W)$ is surjective, and $v_1, \ldots, v_n$ spans V. Prove that the list $Tv_1, \ldots, Tv_n$ spans W.

2. Suppose $T \in \mathcal{L}(V, W)$ is injective, and $Tv_1, \ldots, Tv_n$ is linearly dependent in W. Prove that the list $v_1, \ldots, v_n$ is linearly dependent in V.

3. Suppose $T \in \mathcal{L}(V, W)$, and $Tv_1, \ldots, Tv_n$ is linearly independent in W. Prove that the list $v_1, \ldots, v_n$ is linearly independent in V.

4. Suppose $S \in \mathcal{L}(V, W)$ and $T \in \mathcal{L}(U, V)$ are each injective. Prove that ST is also injective.

5. Suppose U is a subspace of W, and W is a finite-dimensional vector space. Let $S \in \mathcal{L}(U, V)$. Prove that there exists $T \in \mathcal{L}(W, V)$ such that $Tu = Su$ for all $u \in U$.

6. (Graduate students only) Suppose $v_1, \ldots, v_n$ is a linearly dependent list of vectors in V. Suppose that $W \neq \{0\}$. Prove that there exist $w_1, \ldots, w_n \in W$ such that no $T \in \mathcal{L}(V, W)$ satisfies $Tv_i = w_i$ for each $i = 1, \ldots, n$.