Math 4326/5322 **LINEAR ALGEBRA** Spring 2019
Dr. Duval **Main Exercises 3**

due 2pm, Thursday, February 21

1. Prove that if v_1, v_2, v_3 is a basis of V, then

$$v_1, v_1 + v_2, v_1 + v_2 + v_3$$

is also a basis of V.

2. Let

$$U = \{(p \in \mathcal{P}_4(\mathbf{F})\colon p''(5) = 0\}.$$

Find a basis of U, and find a subspace W of $\mathcal{P}_4(\mathbf{F})$ such that $\mathcal{P}_4(\mathbf{F}) = U \oplus W$.

3. Suppose U and W are each 5-dimensional subspaces of $\mathbf{C}^8$. Prove that there are two vectors u, w in $U \cap W$ such that $u \neq zw$ for any $z \in \mathbf{C}$.

4. Suppose U_1, U_2, U_3 are finite-dimensional subspaces of V. Prove that $U_1 + U_2 + U_3$ is finite-dimensional and

$$\dim(U_1 + U_2 + U_3) \leq \dim U_1 + \dim U_2 + \dim U_3.$$

5. (Graduate students only) Assume U and W are subspaces of a finite-dimensional vector space V, and that $U \oplus W = V$. Let $u_1, \ldots, u_n$ be a basis of U and let $w_1, \ldots, w_m$ be a basis of W. Prove that

$$u_1, \ldots, u_n, w_1, \ldots, w_m$$

is a basis of V.