Math 4326/5322 **LINEAR ALGEBRA** Spring 2019

Dr. Duval **Main Exercises 2**

due 2pm, Thursday, February 14

1. Prove or give a counterexample: If $v_1, v_2, \ldots, v_k$ is a linearly independent list of vectors in V, and $c_1, \ldots, c_k \in \mathbf{F}$ with $c_i \neq 0$ for all i, then

$$c_1v_1, c_2v_2, \ldots, c_kv_k$$

is linearly independent.

2. Let $v_1, v_2, \ldots, v_k$ be a list of vectors in V. Prove that

$$\operatorname{span}(v_1, v_2, v_3, \ldots, v_k) = \operatorname{span}(v_1 + v_2, v_1 - v_2, v_3, \ldots, v_k)$$

3. Let

$$U = \{(x, x, y, y, x + y) \in \mathbf{F}^5 \colon x, y \in \mathbf{F}\}.$$

Find a subspace W of $\mathbf{F}^5$ such that $\mathbf{F}^5 = U \oplus W$.

4. Let U and W be subspaces of V. Prove that $U \cap W$ is also a subspace of V.

5. (Graduate students only) Let $\{U_\alpha \colon \alpha \in \Delta\}$ be a collection of subsets in V. Prove that

$$\bigcap_{\alpha \in \Delta} U_\alpha$$

is also a subspace of V.