Math 4326/5322 | **LINEAR ALGEBRA** | Spring 2019

Dr. Duval | **Main Exercises 11**

due 2pm, Thursday, May 2

1. Let V be a finite-dimensional inner product space and let $T \in \mathcal{L}(V)$ be an invertible linear operator. Prove that T^* is also invertible and that $(T^*)^{-1} = (T^{-1})^*$.

2. Suppose n is an integer. Define $T \in \mathcal{L}(\mathbf{F}^n)$ by

$$T(z_1, \ldots, z_n) = (0,\ z_1,\ z_1 + z_2,\ z_1 + z_2 + z_3,\ \ldots,\ z_1 + \cdots + z_{n-1}).$$

Find a formula for $T^*(z_1, \ldots, z_n)$.

3. In $\mathbf{R}^4$, let

$$U = \operatorname{span}((1,1,1,1),(6,1,0,-1)).$$

Find $u \in U$ such that $\|u - (2,0,1,9)\|$ is as small as possible.

4. Let U and W be subspaces of a finite-dimensional inner product space V, and let $P_U, P_W \in \mathcal{L}(V)$ denote the orthogonal projections of V onto U and W, respectively. Prove that if $U \subseteq W$, then

$$P_W P_U = P_U P_W = P_U.$$

5. (Graduate students only) Again let U and W be subspaces of a finite-dimensional inner product space V, and let $P_U, P_W \in \mathcal{L}(V)$ denote the orthogonal projections of V onto U and W, respectively. Prove that if

$$P_U + P_W = I$$

then $U \cap W = \{0\}$.