Math 4326 **LINEAR ALGEBRA** Spring 2017

Dr. Duval **Homework**

Thursday, April 27

Follow the separate general guidelines for Parts A,B,C. Be sure to include and label *all four* standard parts (a), (b), (c), (d) of Part A in what you hand in.

Positive Operators and Isometries (Part II): Isometries

Section 7.C, pp. 228–231

A: Reading questions. Due by 2pm, Mon., 1 May

1. The short paragraph between the definition of isometry (7.37) and Example 7.38 claims that "we will soon see that if $\mathbf{F} = \mathbf{C}$, the the next example includes all isometries." When do we see this?
2. The paragraph above result 7.42 (Characterization of isometries) claims that an operator is an isometry "if and only the list of columns of its matrix with respect to ...some basis is orthonormal". Verify this in the case of the orthonormal basis of $\mathbf{R}^2$, $((3/5, 4/5), (-4/5, 3/5))$. In other words, show that if we make a matrix M whose columns are the two vectors of that basis, then the operator S corresponding to M an isometry. (It is probably easier to do this working with M instead of S, i.e., using the matrix directly.)
3. Continuing the previous question: Verify that S from that question satisfies parts (b) and (h) of result 7.42. (Once again, this may be easier working with M directly.)
4. The paragraph above result 7.43 (Description of isometries when $\mathbf{F} = \mathbf{C}$) says that we need to know that isometries are normal in order to prove result 7.43. Where in that proof do we actually use that S is normal?

B: Warmup exercises. For you to present in class. Due by the end of class Tue., 2 May

Exercises 7.C: none

Polar Decomposition and Singular Value Decomposition

Section 7.D

A: Reading questions. Due by 2pm, Wed., 3 May

1. The paragraph above result 7.45 (Polar Decomposition) claims that "T^*T is a positive operator for every $T \in \mathcal{L}(V)$, and thus $\sqrt{T^*T}$ is well defined." Explain how we know each piece of the sentence is true: How do we know T^*T is a positive operator? And how do we know that $\sqrt{T^*T}$ is well defined, just because T^*T is positive?
2. Verify the calculation at the beginning of Example 7.50, that $T^*T(z_1, z_2, z_3, z_4) = (9z_1, 4z - 2, 0, 9z_4)$ as the textbook says you should. Also verify the claim at the end of that example, that -3 and 0 are the only eigenvalues of T.
3. Perhaps the most surprising thing about result 7.51 (Singular Value Decomposition) is how we need **two** orthonormal bases, $e_1, \ldots, e_n$ and $f_1, \ldots, f_n$. How do we pick each of these bases? (You will need to read the proof somewhat, but I think you do not have to understand all the details yet.)
4. In the proof of result 7.52 (Singular values without taking square root...), why can we use the Spectral Theorem, even though we are **not** assuming T is self-adjoint or normal?
5. What will you do with all the time you have, now that there are no more reading questions to answer?

B: Warmup exercises. For you to present in class. Due by end of class Thu., 4 May

Exercises 7.D: 5