Math 4326 — **LINEAR ALGEBRA** — Spring 2017

Dr. Duval — **Homework**

Thursday, April 20

Follow the separate general guidelines for Parts A,B,C. Be sure to include and label *all four* standard parts (a), (b), (c), (d) of Part A in what you hand in.

The Spectral Theorem

Section 7.B

A: Reading questions. Due by 2pm, Mon., 24 Apr.

1. What is the significance of normal operators and self-adjoint operators, as revealed in this section?
2. Verify all the details of Example 7.23.
3. Compare the **proof** of result 7.27 (Self-adjoint operators have eigenvalues) with the **proof** of result 5.21 (Operators on complex vector spaces have an eigenvalue).
4. Verify the details of Example 7.30.

B: Warmup exercises. For you to present in class. Due by the end of class Tue., 25 Apr.

Exercises 7.B: 2

Positive Operators and Isometries (Part I): Positive Operators

Section 7.C, pp. 225–227

A: Reading questions. Due by 2pm, Wed., 26 Apr.

1. Verify Example 7.32(a).
2. The hardest part of the proof of result 7.35 (Characterization of positive operators) is (b) $\Rightarrow$ (c), but it's not that hard. Where in that part of the proof do we use the hypothesis that T is self-adjoint, and where do we use the hypothesis that all of the eigenvalues of T are nonnegative?
3. Find three different examples of positive operators in $\mathcal{L}(\mathbf{R}^2)$. (Hint: Can you use any part of result 7.35?)
4. In the side note at the beginning of the proof of result 7.36 (Each positive operator has only one positive square root), the textbook claims that the identity operator on V has infinitely many square roots if $\dim V > 1$. Verify this for the case $V = \mathbf{R}^2$ by finding an infinite family of square roots of I. (Hint: Find solutions to the equation $R^2 = I$ by assigning variables to all the entries of the matrix corresponding to R, and then find solutions to the resulting system of equations. You do **not** have to find **all** the solutions.

B: Warmup exercises. For you to present in class. Due by end of class Thu., 27 Apr.

Exercises 7.C: 5