Math 4326 | **LINEAR ALGEBRA** | Spring 2017

Dr. Duval | **Homework**

Tuesday, March 28

Follow the separate general guidelines for Parts A,B,C. Be sure to include and label *all four* standard parts (a), (b), (c), (d) of Part A in what you hand in.

Inner Products and Norms (Part I): Inner Products

Section 6.A, pp. 164–168

A: Reading questions. Due by 2pm, Mon., 3 Apr.

1. The text claims in the middle of p. 164 that "The norm is not linear on $\mathbf{R}^n$." Verify this claim. [Hint: Define a function $N\colon \mathbf{R}^n \to \mathbf{R}$ by $N(x) = \|x\|$, and show N is not linear.] How does this claim relate to the introduction of inner products?
2. Provide a little more explanation for the claim in the middle of p. 165, "The equation above thus suggests that the inner product of $w = (w_1, \ldots, w_n) \in \mathbf{C}^n$ with z should equal
$$w_1\bar{z}_1 + \cdots + w_n\bar{z}_n.\text{"}$$
3. Match the properties of the dot product described at the bottom of p. 164 to the five properties of Definition 6.3 (inner product).
4. Provide justification for each step in the derivation of 6.7(d), that $\langle u, v + w\rangle = \langle u, v\rangle + \langle u, w\rangle$. Note that some of these will be properties of inner products, and others will be properties of complex conjugates (see Definition 4.3).

B: Warmup exercises. For you to present in class. Due by the end of class Tue., 4 Apr.

Verify 6.4(a)-(c) are inner products (satisfy all five conditions of Definition 6.3)

Inner Products and Norms (Part II): Norms

Section 6.A, pp. 168-174

A: Reading questions. Due by 2pm, Wed., 5 Apr.

1. Provide justification for each step in the derivations, of results 6.10(b) and 6.18, respectively, that $\|av\| = |a|\|v\|$, and $\|u + v\| \leq \|u\| + \|v\|$. Note that some of these will be properties of inner products, and others will be properties of complex conjugates (see Definition 4.3).
2. Verify the claim in the middle of p. 171 that the displayed equation above result 6.14 writes u as a scalar multiple of v plus a vector orthogonal to v.
3. Directly verify the Cauchy-Schwarz inequality (result 6.15) for the following pairs of vectors:
   - $(3, 1, 4)$ and $(2, 7, 1)$ in $\mathbf{R}^3$, with inner product 6.4(a); and
   - x^2 and $7x - 2$ in $\mathcal{P}_2(\mathbf{R})$, with inner product 6.4(c).

B: Warmup exercises. For you to present in class. Due by end of class Thu., 6 Apr.

Exercises 6.A: 4