Math 4326 Dr. Duval

## LINEAR ALGEBRA Homework

Tuesday, March 7

Follow the separate general guidelines for Parts A,B,C. Be sure to include and label *all* four standard parts (a), (b), (c), (d) of Part A in what you hand in.

## Invariant Subspaces Section 5.A

A: Reading questions. Due by 2pm, Mon., 20 Mar.

- 1. Why are invariant subspaces important?
- 2. How is equation in Definition 5.5 (eigenvalue) connected with one-dimensional invariant subspaces?
- 3. Does the choice of **F** affect the eigenvalues and eigenvectors of a linear transformation? If so, give an example; if not, explain why not.
- 4. Fill in the missing details of how we get from equation 5.12 to the next (unnumbered) displayed equation,

$$\lambda_k v_k = a_1 \lambda_1 v_1 + \dots + a_{k-1} \lambda_{k-1} v_{k-1}.$$

B: Warmup exercises. For you to present in class. Due by the end of class Tue., 21 Mar.Exercises 5.A: 1, 7, 8

## Eigenvectors and Upper Triangular-Matrices (Part I): Polynomials Applied to Operators and Existence of Eigenvalues Section 5.B, pp. 143-145

## A: Reading questions. Due by 2pm, Wed., 22 Mar.

- 1. Why doesn't  $T^m$  make sense when T is a linear map, but not a linear operator?
- 2. Theorem 5.21 (Operators on complex vector spaces have an eigenvalue) is the most important result of Chapter 5, but its proof, while very clever, is not very complicated at all. Please try your best to understand it. Where in the proof do we use that the vector space is complex (*i.e.*,  $\mathbf{F} = \mathbf{C}$ )?
- 3. What does the proof of Theorem 5.21 have in common with the proof of Theorem 5.10 (Linearly independent eigenvectors)? [Note: Compare the **proofs**, not the statements, of these theorems.]
- B: Warmup exercises. For you to present in class. Due by end of class Thu., 23 Mar. Exercises 5.B: 1, 2