Tuesday, February 21
   Follow the separate general guidelines for Parts A,B,C. Be sure to include and label *all*
four standard parts (a), (b), (c), (d) of Part A in what you hand in.

Invertibility and Isomorphism
Section 3.D

This is a longer section. For now, you only have to read carefully pp. 80–82, 84.

A: Reading questions. Due by 2pm, Mon., 27 Feb.

1. How many inverses can a linear transformation have?

2. When, if ever, in the proof of result 3.56 do we use the linearity of T or of any other map?

3. In the first half of the proof of result 3.59, it is claimed, "Because T is invertible, we have null $T = \{0\}$ and range $T = W$." Why is this implication true?

4. Why define $\mathcal{M}(v)$ as it is defined in Definition 3.62, as opposed to the more simple definition given for $\mathcal{M}(x)$ in Example 3.63?

B: Warmup exercises. For you to present in class. Due by end of class Tue., 28 Feb.

   Exercises 3.D: 1

Duality
Section 3.F

You are only responsible for the subsections "The Dual Space and the Dual Map" and "The Matrix of the Dual of a Linear Map".

A: Reading questions. Due by 2pm, Wed., 1 Mar.

1. Is $\phi\colon \mathbf{R}^2 \to \mathbf{R}$ given by $\phi(x, y) = xy$ a linear functional on $\mathbf{R}^2$? Why or why not?

2. In Definition 3.94, how do we know that V' is actually a vector space?

3. Let's illustrate Definition 3.96. Note that $v_1 = (1, 0, 0), v_2 = (1, 1, 0), v_3 = (1, 1, 1)$ is a basis of $\mathbf{F}^3$ (you don't have to prove this). Apply Definition 3.96 to define ϕ_1, ϕ_2, ϕ_3. What is $\phi_2(9, 1, 5)$? (Hint: Write $(9, 1, 5)$ as a linear combination of v_1, v_2, v_3.)

4. Let's illustrate Definition 3.99. Let $T \in \mathcal{L}(\mathbf{F}^3, \mathbf{F}^2)$ defined by $T(x, y, z) = (x + y + z, 2x - z)$. Let $\phi \in (F^2)'$ be defined by $\phi(u, v) = 3u + 5v$. What is $T'(\phi)$? (Hint: $T'(\phi) \in (F^3)'$.)

5. Verify result 3.114 on the linear map T in question 4 above. (Hint: First find $\phi_1, \phi_2 \in (F^2)'$.)

B: Warmup exercises. For you to present in class. Due by end of class Thu., 2 Mar.

   Exercises 3.F: 4, 12