

Tuesday, February 7

Follow the separate general guidelines for Parts A,B,C. Be sure to include and label *all four* standard parts (a), (b), (c), (d) of Part A in what you hand in.

Null Spaces and Ranges (part I):
Injectivity and Surjectivity
Section 3.B, pp. 59–62

A: Reading questions. Due by 2pm, Mon., 13 Feb.

1. Find the null space of the identity map, defined on p. 53. Is this map injective? Why or why not?
2. Find the range of the multiplication by x^2 map, defined on p. 53. Is this map surjective? Why or why not?
3. Identify, as precisely as you can, when we **use** the linearity of T in the proofs of results 3.14 (null space is subspace), 3.16 (injectivity is equivalent . . .), and 3.19 (range is subspace). [Pinpoint the exact equations and statements that depend on linearity, and which part of the definition of linearity that is used in each case.]
4. Does surjectivity of a map $T \in \mathcal{L}(V, W)$ depend on V , W , both, or neither? If it does depend on V and/or W , give an example showing how changing V and/or W changes the surjectivity of T .

B: Warmup exercises. For you to present in class. Due by end of class Tue., 14 Feb.

Exercises 3.B: 1, 3.

Null Spaces and Ranges (part II):
Fundamental Theorem of Linear Maps
Section 3.B, pp. 63–66

A: Reading questions. Due by 2pm, Wed., 15 Feb.

1. In the proof of Theorem 3.22 (Fundamental Theorem of Linear Maps), what are m and n , and how do we know $\dim V = m + n$? How do we compute $\dim \text{null } T$ and $\dim \text{range } T$? [Note: Theorem 3.22 is the most important theorem of the first four chapters of the book, and also has one of the longest proofs in these chapters. You can answer these reading questions just from carefully reading and understanding the first paragraph of the proof, which is all I ask you to do, though, of course, you are welcome to read the rest of the proof.]
2. In the proof of result 3.23, there is a string of equalities and inequalities. The middle line of this string reads “ $\geq \dim V - \dim W$ ”. Explain why “ \geq ” is the correct relation here.
3. In Example 3.25, the text claims the equation $T(x_1, \dots, x_n) = 0$ “is the same as the homogeneous system of linear equations above.” Why are they the same?

B: Warmup exercises. For you to present in class. Due by the end of class Thu., 16 Feb..

Exercises 3.B: 5, 6, 13.