Math 4326 **LINEAR ALGEBRA** Spring 2015
Dr. Duval **Homework**
Monday, April 20

Follow the separate general guidelines for Parts A,B,C. Be sure to include and label *all four* standard parts (a), (b), (c), (d) of Part A in what you hand in.

Decomposition of an Operator

Section 8.B

We are not discussing the subsection on Square Roots at the end of this section.

A: Reading questions. Due by 2pm, Sun., 26 Apr.

1. Justify each equality in the first displayed string of equalities in the proof of result 8.20 (The null space and range. . .):

$$((p(T))(Tv) = T(p(T)v) = T(0) = 0.$$

   Any step you can't justify, demonstrate with a small example (say, where p is a quadratic polynomial, and T is an operator on $\mathbf{F}^2$).

2. Demonstrate all three parts of result 8.21 (Description of operators. . .) on the linear operator T in Example 8.12. The proof of result 8.21 may give some hints, though you don't need to understand all the details of this proof to answer this question. It **may** also help to use the matrix representation of T and other operators.

3. Demonstrate result 8.23 (A basis of generalized eigenvectors) on the linear operator T in Example 8.12.

4. Find the multiplicities of each of the eigenvalues of the linear operator T in Example 8.12.

5. Prove the **converse** of result 8.29 (Block diagonal matrix. . .), that the eigenvalues of any block diagonal matrix whose blocks have the form shown in the matrices in the statement of the theorem are $\lambda_1, \ldots, \lambda_m$. Construct a **good** example of a 7-by-7 matrix of this form, with eigenvalues 3, 0, and -1. (Here, "good" means with as few zero's as possible.)

B: Warmup exercises. For you to present in class. Due by the end of class Mon., 27 Apr.

Exercises 8.B: 1

Characteristic and Minimal Polynomials (Part I)

Section 8.C, pp. 261–264

A: Reading questions. Due by 2pm, Tue., 28 Apr.

1. Find the characteristic polynomial of the linear operator whose matrix is given in Example 8.28. Verify that this characteristic polynomial satisfies result 8.36 (Degree and zeros…).
2. Verify the linear operator defined in Example 8.25 satisfies result 8.3 (Cayley-Hamilton Theorem).
3. According to the definition of minimal polynomial (Definition 8.43), the linear operator T whose matrix is given in Example 8.45 should satisfy $p(T) = 0$ for the polynomial $p(z) = z^5 - 6z + 3$. Verify this. (Note, you do **not** have to verify that p is the *minimal* polynomial of T, you only have to verify that $p(T) = 0$.)
4. In result 8.40, why do we insist the polynomial is **monic**? What would happen if we drop that assumption.

B: Warmup exercises. For you to present in class. Due by end of class Wed., 29 Apr.

Exercises 8.C: 3