Math 4326 | **LINEAR ALGEBRA** | Spring 2015

Dr. Duval | **Homework**

Monday, April 13

Follow the separate general guidelines for Parts A,B,C. Be sure to include and label *all four* standard parts (a), (b), (c), (d) of Part A in what you hand in.

Generalized Eigenvectors

Section 8.A, pp. 242–247

A: Reading questions. Due by 2pm, Sun., 19 Apr.

1. Verify result 8.3 for the linear operator $T \in \mathcal{L}(\mathbf{F}^4)$ given by $T(z_1, z_2, z_3, z_4) = (z_1, z_3, z_4, 0)$.
2. In Example 8.7 show why $T^3(z_1, z_2, z_3) = (0, 0, 125z_3)$.
3. The text states, at the top of p. 245, that the operator in 5.43 "does not have enough eigenvectors for 8.8 to hold." Explain carefully what that means in this case.
4. The text claims, in the margin of p. 245, that "if $(T - \lambda I)^j$ is not injective for some positive integer j, then $T - \lambda I$ is not injective ...". Verify this claim.
5. What is a generalized eigenvector, and why do we call it that?

B: Warmup exercises. For you to present in class. Due by the end of class Mon., 20 Apr.

Exercises 8.A: 1

Nilpotent Operators

Section 8.A, pp. 248–249

A: Reading questions. Due by 2pm, Tue., 21 Apr.

1. Verify the claim Example 8.17 that the operator $N \in \mathcal{L}(\mathbf{F}^4)$ defined by $N(z_1, z_2, z_3, z_4) = (z_3, z_4, 0, 0)$ satisfies $N^2 = 0$.
2. Find a linear operator in $\mathcal{L}(\mathbf{F}^4)$ that is **not** nilpotent, and show it is not nilpotent.
3. Explain more carefully the following claim made at the beginning of the proof of result 8.18: "Because N is nilpotent, $G(0, N) = V$."
4. Give three different examples of 3×3 matrices corresponding to nilpotent operators. Use matrix multiplication to verify that one of them is nilpotent. Use as few zeros in your matrices as you can.

B: Warmup exercises. For you to present in class. Due by end of class Wed., 22 Apr.

Exercises 8.A: 7