Math 4326 | **LINEAR ALGEBRA** | Spring 2008
Dr. Duval | **Homework**

Tuesday, April 15

Follow the separate general guidelines for Parts A,B,C. Be sure to include and label *all four* standard parts (a), (b), (c), (d) of Part A in what you hand in.

Generalized Eigenvectors (part II)

pp. 167–168

A: Reading questions. Due by 2pm, Mon., 21 Apr.

1. Verify the claim at the top of p. 167 that the operator $N \in \mathcal{L}(\mathbf{F}^4)$ defined by $N(z_1, z_2, z_3, z_4) = (z_3, z_4, 0, 0)$ satisfies $N^2 = 0$.
2. Find a linear operator in $\mathcal{L}(\mathbf{F}^4)$ that is **not** nilpotent, and show it is not nilpotent.
3. Explain more carefully the following claim made at the beginning of the proof of Corollary 8.8: "Because N is nilpotent, every vector in V is a generalized eigenvector corresponding to the eigenvalue 0."
4. Verify both Proposition 8.9 and the displayed equation above it, $V = \operatorname{range} T^0 \supset \operatorname{range} T^1 \supset \cdots \supset \operatorname{range} T^k \supset \operatorname{range} T^{k+1}$, for the linear operator $T \in \mathcal{L}(\mathbf{F}^4)$ given by $T(z_1, z_2, z_3, z_4) = (z_1, z_3, z_4, 0)$.

B: Warmup exercises. For you to present in class. Due by end of class Tue., 22 Apr.

Ch. 8: Exercises 5, 6.

The Characteristic Polynomial (part I)

pp. 168–171

[Part I covers through the end of the proof of Theorem 8.10.]

A: Reading questions. Due by 2pm, Wed., 23 Apr.

1. Answer the question posed in the middle of p. 168, "Could the number of times that a particular eigenvalue is repeated depend on which basis of V we choose?"
2. Demonstrate Theorem 8.10 on on the 4-by-4 upper triangular matrix near the top of p. 83. In other words, show that $\dim \operatorname{null}(T - \lambda I)^{\dim V}$ is 2 for $\lambda = 6$, since 6 appears twice on the diagonal, and is 1 for $\lambda = 7, 8$, since 7 and 8 each appear once on the diagonal. Note that the basis here is the standard basis.
3. Demonstrate the claim, made in the margin of p. 168, that if T has a diagonal matrix A with respect to some basis, then λ appears on the diagonal of A precisely $\dim \operatorname{null}(T - \lambda I)$ times, on the linear operator $T \in \mathcal{L}(\mathbf{F}^3)$ defined by $T(z_1, z_2, z_3) = (4z_1, 4z_2, 5z_3)$ on p. 88. Note that the basis here is the standard basis. Why is this claim a special case of Theorem 8.10?

B: Warmup exercises. For you to present in class. Due by end of class Thu., 24 Apr.

Ch. 8: 10.