Math 4326 **LINEAR ALGEBRA** Spring 2008
Dr. Duval **Homework**
Thursday, March 6

Follow the separate general guidelines for Parts A,B,C. Be sure to include and label *all four* standard parts (a), (b), (c), (d) of Part A in what you hand in.

Polynomials Applied to Operators
Upper-Triangular Matrices (part I)
pp. 80–83

A: Reading questions. Due by 2pm, Wed., 12 Mar.

1. Why doesn't T^m make sense when T is a linear map, but not a linear operator?
2. Theorem 5.10 is the most important result of Chapter 5, but its proof, while very clever, is not very complicated at all. Please try your best to understand it. Where in the proof do we use that the vector is complex (*i.e.*, $\mathbf{F} = \mathbf{C}$)?
3. What does the proof of Theorem 5.10 have in common with the proof of Theorem 5.6? [Note: Compare the **proofs**, not the statements, of these theorems.]
4. Give three examples of a 3-by-3 upper-triangular matrix.
5. The author introduces upper-triangular matrices as nice ones to represent linear transformations. Given a fixed linear transformation (in other words, you don't get to pick the linear transformation), how can you represent it by an upper-triangular matrix? In other words, *what* can you pick cleverly to make sure the fixed linear transformation is represented by an upper-triangular matrix?

B: Warmup exercises. For you to present in class. Due by end of class Thu., 13 Mar.

Ch. 5: Exercises 9, 10.

Upper-Triangular Matrices (part II)
pp. 83–87

A: Reading questions. Due by 2pm, Mon., 17 Mar.

1. Demonstrate parts (b) and (c), for $k = 3$, of Proposition 5.12 on the 4-by-4 upper triangular matrix near the top of p. 83. In other words, show that $Tv_3 \in \operatorname{span}(v_1, \ldots, v_3)$ and that $\operatorname{span}(v_1, \ldots, v_3)$ is invariant under T. Note that the basis here is the standard basis.
2. In the middle of the second paragraph of the proof of Theorem 5.13, the text claims that, if λ is an eigenvalue of T, then $T - \lambda I$ is not surjective by (3.21). Fill in the missing details of this claim.
3. Let $T \in \mathcal{L}(V)$. How does finding a basis of V for which the matrix of T is upper triangular help find the eigenvalues of T? How does it help determine whether or not T is invertible? Demonstrate your answers on the 4-by-4 upper triangular matrix near the top of p. 83 (in other words, find the eigenvalues of that matrix, and also determine whether or not it is invertible).

B: Warmup exercises. For you to present in class. Due by end of class Tue., 18 Mar.

Ch. 5: Exercises 18, 19.