Math 4326 **LINEAR ALGEBRA** Spring 2008
Dr. Duval **Homework**

Tuesday, January 22

Follow the separate general guidelines for Parts A,B,C. Be sure to include and label *all four* standard parts (a), (b), (c), (d) of Part A in what you hand in.

Sums and Direct Sums

pp. 14–18

A: Reading questions. Due by 5pm, Wed. 23 Jan., if possible; beginning of class Thu. 24 Jan., otherwise.

1. Verify that $U_1 + \cdots + U_m$ is a subspace.
2. Verify equation 1.7. [Note that the text uses 1.7 twice, each with a different choice of W. Pick *one* choice of W to verify equation 1.7 for.]
3. Does $U + W$ exist for *any* pair of subspaces U and W? Does $U \oplus W$ exist for *any* pair of subspaces U and W? Justify your answer in each case.
4. Verify $\mathbf{F}^3 = U \oplus W$ in the example in the middle of p. 15.
5. In the proof of Proposition 1.8, where do we use the assumption that the U_i's are subspaces? [Note: This may be at just one point in the proof, or at more than one point.]
6. Which do you think will be more useful for establishing a direct sum equation, Proposition 1.8, or Proposition 1.9? Justify your answer. If you cannot decide, then present arguments in favor of both.

B: Warmup exercises. For you to present in class. Due by end of class Thu., 24 Jan.

Ch. 1: Exercises 10, 11, 13.

Span and Linear Independence

pp. 22–27

A: Reading questions. Due Mon. 28 Jan., 2pm.

1. Verify that the span of any list of vectors in V is a subspace of V.
2. Why should the span of an empty list be $\{0\}$ [the vector space whose only vector is the 0 vector]?
3. Verify that, if some vectors are removed from a linearly independent list, then the remaining list is also linearly independent.
4. Why is the empty list linearly independent?
5. Demonstrate Lemma 2.4 on the linearly dependent list from the middle of p. 24, $((2, 3, 1), (1, -1, 2), (7, 3, 8))$. In other words, find the v_j that makes (a) and (b) true, and show why (a) and (b) are in fact true in this case. [Hint: Use the proof.]
6. Demonstrate the multistep process described in the proof of Theorem 2.6 on the linearly independent list $((1, 1, 1), (1, 2, 0))$ and the linearly dependent list $((2, 3, 1), (1, -1, 2), (7, 3, 8))$.

B: Warmup exercises. For you to present in class. Due by end of class Tue., 29 Jan.

Verify that $\mathcal{P}_m(F)$ is a subspace of $\mathcal{P}(F)$.

Ch. 2: Exercises 1, 2, 4.