3 Homework 3

Exploring Power and Logs

Justify all steps with power properties and log properties.

1. Find the following:
   (a) $\log_3(\sqrt[3]{9})$
   (b) $\log_9(\sqrt[3]{9})$
   (c) $\log_{\frac{1}{3}}(\sqrt[3]{9})$
   (d) $\log_{\sqrt{3}}(\sqrt[3]{9})$
2. Find the following:
   (a) $\log_5 \sqrt{5}$
   (b) $\log_{\sqrt{5}} 5$
3. Find $\log_{10^n} 0.01$. Explain your reasoning.
4. Solve for n:
$$(1+(2+\sqrt{n})^{\frac{1}{2}})^{\frac{1}{2}} = 3.$$
5. Solve for x in $3^{(81^x)} = 81^{(3^x)}$.
6. (a) Solve for b: $\log_{2b} 216 = b$.
   (b) If $\log_{\frac{1}{2}} a = L$, what is $\log_2 a$?
   (c) If $\log_{2^y} P = L$, what is $\log_2 P$?
7. Let $a, b > 0$ be different real numbers, and let $u, v, w, x \in \mathbb{R}$. Suppose that
$$a^u = b^w, \qquad b^v = a^x. \qquad (*)$$
   (a) What are some examples of a, b, u, v, w, x that satisfy the relationship (*)? Come up with 5 examples.
   (b) Conjecture a relationship between u, v, w, x that holds regardless of the value of a, b.
   (c) Prove the relationship.
8. Let $a \in \mathbb{R}_{>0}, a \neq 1$. Prove that logarithm base a satisfies the Log of Product rule. (Remember that part of your write-up includes a precise statement of the Log of Product rule.)

Exponential Growth is Defined by Constant Change

We have defined 1-unit and n-unit change factors in the context of exponential functions, but they can be applied to other functions as well. We will explore in some of the questions below.

Definition. Given a function $f : \mathbb{R} \to \mathbb{R}$, and $x \in \mathbb{R}$ in the domain of f:

- We say that the **1-unit change factor at** x is the ratio $f(x+1)/f(x)$.
- Let $n \in \mathbb{R}$. We say that the ***n*-unit change factor at** x is the ratio $f(x+n)/f(x)$.

9. Let $f(x) = -2x$. Use the definition of change factor shown above. What is the 1-unit change factor of f at …
   (a) … $x = 0.1$?
   (b) … $x = 1$?
   (c) … $x = 10$?
   (d) … $x = 100$?

10. Let $f(x) = x^2$. Use the definition of change factor shown above. What is the 1-unit change factor of f at ...
    (a) $\ldots x = 0.1$?
    (b) $\ldots x = 1$?
    (c) $\ldots x = 10$?
    (d) $\ldots x = 100$?
11. Answer and explain:
    (a) What are all possible 1-unit change factors for $f(x) = -2x$, for $|x| \leq 1$?
    (b) What are all possible 1-unit change factors for $f(x) = -2x$, for $|x| \geq 1$?
12. Answer and explain:
    (a) What are all possible 1-unit change factors for $f(x) = x^2$, for $|x| \leq 1$?
    (b) What are all possible 1-unit change factors for $f(x) = x^2$, for $|x| \geq 1$?
13. Answer and explain: Which of the following functions have constant change factors?
    (a) $f(x) = 3^{4x} - 5$
    (b) $f(x) = 3^{4x-5}$
    (c) $f(x) = \frac{3^{4x}}{5}$
    (d) $f(x) = 3^{4x}5^x$

    In your explanations:
    - Give numerical examples to support your explanation.
    - If it does have constant change factors, express it in the form ba^x.
14. Let $f(x) = 3^{4x-5}$.
    (a) What are all possible 1-unit change factors of f?
    (b) What are all possible $\frac{1}{2}$-unit change factors of f?
    (c) What are all possible π-unit change factors of f?
15. Find all functions f such that they have constant change factors and $f(0) = 1$.
    (a) Express your answer algebraically.
    (b) Represent your answer graphically.

Topics in Exponential Expressions and Functions

16. In January 2010, the price of tea in Flotinnia was $81.80 per pound.1 Each year, the price of tea in Flotinnia increased by 3.4% of its value the previous year.
    (a) Write an expression for the price of tea per pound in Flotinnia in January 2011. Is it possible to write this expression so that it has only one term (that is, so that the expression is not a sum of multiple terms)?
    (b) Write expressions for the price of tea per pound in Flotinnia in January 2012 and January 2013
    (c) Let $p(t)$ represent the price in dollars of a pound of tea in Flotinnia t years after January 2010. Write an equation that shows how $p(t)$ is related to $p(t-1)$ for a given value of t. (Such an equation is called a recursive formula for $p(t)$, since it defines $p(t)$ in terms of other function values.)
    (d) Now, write a formula for $p(t)$ in terms of t only; that is, without referring to $p(t-1)$ or any other value of the function p. (Such a formula is called an explicit formula for $p(t)$, since it defines $p(t)$ directly in terms of the input variable t.)
    (e) Explain how a student unfamiliar with exponential functions could come up with the formula you wrote in part (d) as a natural generalization of the calculations performed in parts (a) and (b).

17. (a) Solve for x and express your responses as an interval.

$$\log_{\frac{1}{2}} 3x < \log_2(3 - 2x)$$

(b) Solve the question in a different way.

(c) Suppose you use these two solutions as example with your class. What would you want them to learn from these examples?