Homework for Chapters 3-4

0. In these chapters, we learned about:

    - Functions and invertible functions
    - Partial inverse of a function and how to construct them
    - Correspondence and covariation views for inverse and composition
    - The mathematical/teaching practices of:
        - Introducing a definition
        - Explaining a mathematical "test" of a property
        - Noticing student thinking
        - Recognizing and explaining correspondence and covariation views.

    For each of these ideas:

    (a) Where in the text are these ideas located?

    (b) Review this section of the text. What definitions and results were important? How do examples use these definitions and results?

    (c) What questions or comments do you have about the ideas in this section?

CHAPTER 3

1. Explain why the following procedure works:

    > To find the formula for the inverse of an invertible function, switch the y's and x's then solve for y.

    Explain why this procedure works in terms of the example:

    (a) $f(x) = 5x$

    (b) $f(x) = x^3 - 1$

    (c) $f(x) = \frac{1}{x-3}$

    Then:

    (d) Explain why this procedure in general terms.

    (e) Explain why this procedure is equivalent to reflecting the graph of the function about the line $y = x$.

    (f) Explain what would work and what would not work if you were to use this procedure on non-invertible functions.

2. (This problem comes from Mason, Burton, and Stacey (2010, p. 203)[3]) Under what conditions can you rotate the graph of a function about the origin, and still have the resulting graph being the graph of a function? If the graph of a function cannot be rotated about the origin without ceasing to be the graph of a function, might there be other points which could act as center of rotation and preserve the property of being the graph of a function?

3. (a) Let f and g be two invertible functions. Explain why $g \circ f$ is invertible in terms of the middle school and university versions of the definition of relation.

    (b) Explain why the inverse of $g \circ f$ should be $(f^{-1}) \circ (g^{-1})$ in terms of the middle school version of the definition of composition.

    (c) Explain why the inverse of $g \circ f$ should be $(f^{-1}) \circ (g^{-1})$ in terms of the university version of the definition of invertible function.

4. (a) Construct three candidates for partial inverses for the sine function.

    (b) On what subset of the domain of sine do each of your candidates serve as a true inverse? On what subset of the domain of sine do your candidates serve as only a partial inverse?

    (c) Construct three candidates for partial inverses for the cosine function?

[3]Mason, J., Burton, L., Stacey, K. (2010). *Thinking Mathematically*. Essex, England: Pearson Education Limited.

(d) On what subset of the domain of cosine do each of your candidates serve as a true inverse? On what subset of the domain of cosine do your candidates serve as only a partial inverse?

5. Explain the following tests of a mathematical property using the structure discussed in Section 3.3.2 (beginning p. 34). In communicating your explanation, place the different sections of the structure separately from each other and label them.

   (a) Vertical line test.

   (b) Horizontal line test.

   For the following tests of mathematical properties, in the section on *why* it works, provide an explanation in two parts, first in terms of a specific example, and second as a general explanation.

   (c) Testing points on a graph of a linear inequality to see which side to shade (e.g., `http://www.wtamu.edu/academic/anns/mps/math/mathlab/beg_algebra/beg_alg_tut24_ineq.htm`). *Note:* The definition of graph of an inequality in x and y is similar to that of graph of an equation: It is the set of points (a, b) that if you evaluate the inequality at $x = a$ and $y = b$, you get a true statement. The property being tested here is of a half-plane, and whether the points in that half-plane satisfy a given inequality.

   (d) Direct comparison test (for convergence of a series).

   (e) Ratio test (for convergence of a series).

6. Examine the table to the right. Based on the given information, discuss how change in x appears to impact change in y.

   (a) If x changes by ± 1 units, how does y appear to change?

   (b) If x changes by $\pm h$ units, how does y appear to change, in terms of h?

   (c) Based on how change in x impacts change in y, and using the given data, describe how you would find a plausible value of y when x is 0.

x	y
3	16
5	26
6	31
9	46

7. Pot A and Pot B have the radial cross section shown below. (This means that to get the shapes of Pot A and Pot B, you can rotate this cross section around a central axis.) The sides of Pot A are vertical. Both pots have a 1 gallon capacity.

   (a) Water is being poured into Pot A at an unsteady pace. Draw a graph that represents the relationship between volume of water and height of the water, with volume as input variable, height as output variable.

   (b) Draw a graph that represents the same relationship for Pot A, but this time with height as an input variable and volume as output variable.

   (c) Water is being poured into Pot B at an unsteady pace. Draw a graph that represents the relationship between volume of water and height of the water, with volume as input variable, height as output variable.

   (d) Draw a graph that represents the same relationship for Pot B, but this time with height as an input variable and volume as output variable.

Pot A Pot B