Homework for Chapter 2

0. In this chapter, we learned about:

   - Defining:
     - Relation, in three equivalent ways; candidate domain, domain, candidate range, range, preimage of a point and of a subset, image of a point and of a subset
     - Inverse relation
     - Composition of relations
     - Graph of a (real) relation; x- and y-intercepts; intersection of graphs
   - Showing that a point (x, y) is on the graph of a relation
   - Showing that a point (x, y) is on the graph of an equation
   - The mathematical/teaching practices of:
     - Connecting mathematically equivalent definitions
     - Connecting different mathematical representations

   For each of these ideas:

   (a) Where in the text are these ideas located?

   (b) Review this section of the text. What definitions and results were important? How do examples use these definitions and results?

   (c) What questions or comments do you have about the ideas in this section?

1. Describe the following concepts in terms of the middle school and university versions of the definition of relation:

   (a) candidate domain, domain

   (b) image of a point, image of a subset

   (c) candidate range, range

   (d) preimage of a point, preimage of a subset

   (e) x- and y-intercepts

   (f) intersection of the graphs of two relations

   (g) inverse relation

   (h) composition of two relations

   In your work for each,

   (i) address both the middle school and university versions of the definition of relation

   (ii) include diagrams

   (iii) describe the concepts using a specific example, using a relation such as the parent relation P or angle relation A as defined in Example 2.5.

   (iv) then phrase the descriptions so that they can apply without any changes to any possible example of a relation

   (v) explain why the descriptions in terms of both versions are mathematically equivalent

In questions 2, 3, and 9, we examine relations which are functions. This means that each input in the domain is assigned to only one output element, so the notation $f(x)$ makes sense: there is only one element that f sends x to. We will also examine this definition more in question 11.

2. (a) Graph the following and their inverse relations. (You may need to look up the relations in a high school resource online or elsewhere.) We will use these examples in the next chapter.

   - sine function ($x \mapsto \sin(x)$ for $x \in \mathbb{R}$)
   - cosine function ($x \mapsto \cos(x)$ for $x \in \mathbb{R}$)
   - absolute value function ($\text{Abs} : \mathbb{R} \to \mathbb{R}, x \mapsto |x|$)

- squaring function ($\mathrm{Sq} : \mathbb{R} \to \mathbb{R}, x \mapsto x^2$)
- cubing function ($\mathrm{Cu} : \mathbb{R} \to \mathbb{R}, x \mapsto x^3$)

(b) In each of the graphs of the given functions, label any maxima or minima with its coordinate. In the graphs of their inverses, label any points that are leftmost or rightmost with its coordinate. Use exact form (this means to not use approximations, including decimal approximations; use symbols such as π as needed).

(c) What are the following images and preimages?

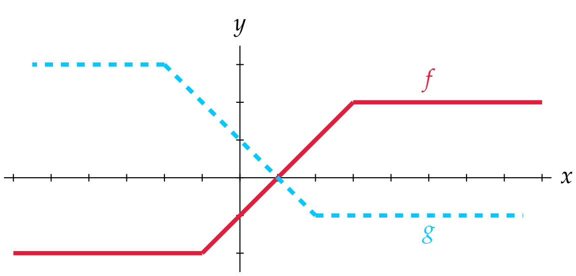
i. What is the image of $[0, \pi]$ for the sine function? How about for the cosine function?
ii. What is the preimage of $[0.5, 1]$ for the sine function? How about for the cosine function?
iii. What is the image of $[0.5, 1] \cup [-3, -1] \cup [3, \infty]$ for Abs?
iv. What is the preimage of $[100, 144]$ for Abs?
v. What is the image of $[0.5, 1] \cup [-3, -1] \cup [3, \infty]$ for Sq?
vi. What is the preimage of $[100, 144]$ for Sq?
vii. What is the image of $[0.5, 1] \cup [-3, -1] \cup [3, \infty]$ for Cu? (for this question, you may use approximations)
viii. What is the preimage of $[100, 144]$ for Cu? (for this question, you may use approximations)

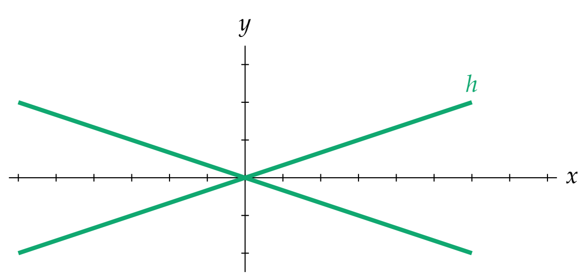
3. Below are graphs of the relations f, g, h. The pieces of these graphs are lines and line segments, and their turning points are integer coordinate points. Consecutive tick marks on the axes are distance 1 from each other.

(a) Find the values of $g \circ f(-4)$, $g \circ f(-1)$, $g \circ f(0)$, $g \circ f(3)$, $g \circ f(4)$.

(b) Where does $h \circ g$ map each of $0, 2, 4$?

(c) Where does $g \circ h$ map each of $0, 2, 4$?





4. In this task, you will watch a short video of teaching by Ms. Barbara Shreve of San Lorenzo High School. The video shows her teaching an intervention class called Algebra Success. The students in this class have been previously unsuccessful in Algebra 1. They are working on finding intercepts of equations to get ready for working with quadratics.

As you watch the video, it may be tempting to think about what you personally think is good or not as good about the teaching, or what you might have done differently. But before getting to these kinds of judgments, it is more important to simply observe what is going on, what the students' reasoning is, and what the story line is. (This is just like when working with students, as we will see later in this class and you will learn in your methods class: before evaluating students' work, we must first observe and understand students' work without judgment.) Here, we will practice observing the interactions between teachers and

As you watch the video, think about the following questions:

(a) How does the teacher emphasize to students to explain their reasoning?

(b) How does the teacher help students feel comfortable sharing their reasoning?

(c) How was the definition of x-intercept or y-intercept used?

Here is a link to the video: `http://www.insidemathematics.org/classroom-videos/public-lessons/9th-11th-grade-math-quadratic-functions/introduction-part-b`

Talk through your responses to the discussion questions with a fellow teacher in our class. As you talk, be sure to point to the evidence that you are basing your ideas on. Then write down your response to these discussion questions. Describe the evidence you are using by selecting quotes from the transcript (found on the video site).

5. (This task can be thought of as a followup to question 4.)
Suppose that you are teaching about intercepts of graphs and you are going over a solution to the problem: *Find the y-intercept of the graph of the equation* $y = (x-3)^2$. Your class has the following conversation:

   You: How did you start this problem?
   Student A: I put in a 0 for the x-value.
   You: Let's talk about what Student A did. If we're finding a y-intercept, why do we start by putting a 0 for the x? Anyone have an idea?
   Student B: Because zero's the easiest thing.
   Student C: Because zero is where the line crosses.
   Student B: Wait, it's because you want to cancel it out.

   (a) Solve the problem that the class is working on.
   (b) Explain the logic of student B's thinking.
   (c) Explain the logic of student C's thinking.
   (d) In the equivalent of at most 4 tweets, explain why " If we're finding a y-intercept, we start by putting a 0 for the x.". (A tweet is 140 characters.) Your explanation should tie together the definition of graph and the definition of y-intercept.
   (e) What questions might you pose to students to get at the ideas in this explanation?

6. Suppose you are teaching about how to find the intersection of graphs of quadratic and linear equations, and you plan for your class to work on this exploratory task:

   How many points do the graphs of $y = x^2$ *and* $y = x$ *intersect at? What are those points?*
   How many points do the graphs of $y = x^2 + a$ *and* $y = x$ *intersect at? What are those points? How does the answer depend on* a*?*

   An important initial step of planning to teach a task is to solve that task yourself.

   (a) Keeping in mind what we have learned about satisfying answers to mathematical questions: What is a good answer to this task? Describe your conjecture.
   (b) Prove your conjecture.

   Another step in planning is to figure out how you might explain how key ideas are used to solve the task.

   (c) Explain as you would to high school students in this class how the definition of intersection of graph is used in finding solutions to the exploration.

7. Show that for any relation $r : D \to D$, if $x \in D$ is in the domain of r, then (x, x) lies on the graph of $r^{-1} \circ r$.
8. (Based on Chazan (1993)[2]). Suppose you are teaching algebra and a student asks, " Why do we call 'x' a variable in equations like $6x + 5 = 10$ when it stands for just one number?"

   (a) What is the student thinking? How might they have arrived at this question?
   (b) What are you sure that the student understands? What are you unsure that the student understands?
   (c) What is the mathematical issue here?
   (d) How do you respond?

9. Suppose that you are introducing the idea that one way to obtain the graph of an inverse of a relation is to reflect the graph of the relation over the line $y = x$.

   (a) To help students understand this reflection, you use examples such as "Where would the point $(3, 0)$ go if you reflected it about the line $y = x$? What about the point $(0, 15)$? $(3, 15)$?" You then follow up with other examples.
   Using the examples $(3, 0)$, $(0, 15)$, and $(3, 15)$, explain geometrically why it makes sense that point $(3, 0)$ is mapped to the point $(0, 3)$ by reflection over the line $y = x$, and similarly for $(0, 15)$ to $(15, 0)$, and $(3, 15)$ to $(15, 3)$.
   Then explain, why, in general, the point (a, b) is mapped to the point (b, a) when reflected over the line $y = x$. This part of your explanation should apply to any possible example of a point without referring specifically to any examples.

[2] Chazan, D. (1993). F(x)=G(x)?: An approach to modeling with algebra. *For the Learning of Mathematics, 13*, 22-26.

(b) You then use the example of $f(x) = 5x$. What is the equation of its inverse relation f^{-1}?

(c) Using the example of $f(x) = 5x$, explain why it makes sense that f^{-1} must have the graph of the equation you found in (b). In your explanation, draw on the definition of inverse relation, the definition of graph of a relation, and the definition of graph of an equation, as well as your general explanation in part (a). Be explicit about where you use these definitions.

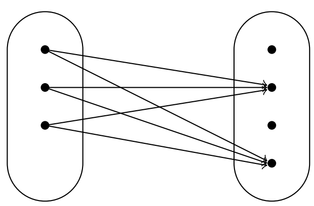
(d) Then explain why, in general, it is true that one way to obtain the graph of an inverse of a relation is to reflect the graph of the relation over the line $y = x$.

10. We can think of definite integrals such as $\int_0^x t^2\,dt$ as a function with input variable x.
Using the definition of graph of an equation, explain why when you shift every point on the graph of $y = \int_0^x t^2\,dt$ down by $\frac{1}{3}$, you obtain the graph of $y = \int_1^x t^2\,dt$.

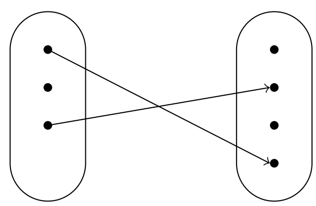
11. Read the following definition:
Definition. A *function* from D to R is defined as a relation from D to R where each input in D is assigned to no more than one output in R.
Complete the following table by placing checkmarks to indicate: which of the relations below are also functions?

	r	r^{-1}	s	s^{-1}	t	t^{-1}
is a function						
is NOT a function						

r



s



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