Homework for Chapter 1

0. In this chapter, we learned about:
   - Showing that an element is or is not a member of a particular set
   - Showing that a set is a subset or strict subset of another set
   - The definition of set equality and how to show that two sets are equal
   - Number parents and children, in particular, that if
     $S = \{n \in \mathbb{N} : n \text{ has at least three different non-trivial factors}\}$ and
     $T = \{n \in \mathbb{N} : n \text{ has more than one pair of parents}\}$, then $S = T$.

   For each of these ideas:

   (a) Where in the text are these ideas located?

   (b) Review this section of the text. What definitions are most relevant? How do the examples use these definitions?

   (c) What questions or comments do you have about the ideas in this section?

1. Let $S = \{x \in \mathbb{Q} : x \text{ can be written as a fraction with denominator 2 and } |x| < 2\}$.
   Let $B = \{1 + 3 + \cdots + (2n+1) : n \in \mathbb{N}\}$.
   Let C be the set of functions of the form $x \mapsto 27^{ax}$.
   Let D be the set of numbers with more than 5 parents.

   (a) Is "$0.25 \in S$" a true statement?

   (b) Is "$1 \in S$" a true statement?

   (c) Is 25 an element of B?

   (d) Is 24 an element of B?

   (e) When $a \in \mathbb{Z}$, is $x \mapsto 3^{5x}$ contained in C?

   (f) When $a \in \mathbb{Q}$, is $x \mapsto 3^{5x}$ contained in C?

   (g) Find a number that is a member of both B and D.

   After reading this problem, think about: What mathematical idea(s) listed in Problem 0 does this problem provide opportunities to understand? This is something helpful to think about for all the problems.
   In your responses, articulate clearly:
   - Whether you are showing that the element is or is not contained in the set;
   - How you used the definition of set membership to determine "yes" or "no"; and
   - Any definitions or ideas that you need in your reasoning.

2. (a) Complete this table with "True" or "False".

	$A \subseteq B$	$A \subsetneq B$	$A \supseteq B$	$A \supsetneq B$	$A = B$	$A \neq B$	Neither is subset of the other
$A =$ integer multiples of 14, $B =$ integer multiples of 21							
$A = \{n^2 : n \in \mathbb{N}, n \geq 1\}$, $B = \{1 + 3 + \cdots + (2n-1) : n \geq 1\}$							
$A =$ functions of the form $x \mapsto 27^{ax}$ $B =$ functions of the form $x \mapsto 3^{ax}$, $a \in \mathbb{Z}$							
$A =$ functions of the form $x \mapsto 27^{ax}$ $B =$ functions of the form $x \mapsto 3^{ax}$, $a \in \mathbb{Q}$							

   (b) Prove your "true" responses to Row 1.

(c) Prove your "true" responses to Row 2.

(d) Prove your "true" responses to Row 3.

(e) Prove your "true" responses to Row 4.

3. Read over the section containing Conjectures 1.3 and 1.4 and Proposition 1.5.

   (a) Write the following statements in your own words: Conjecture 1.3 and Conjecture 1.4. Then explain why they are mathematically equivalent.

   (b) In the proof of Proposition 1.5, in the section on showing $T \subseteq S$, the proof states, "Let $n \in T$. Then there exist at least two pairs $a, a' \in \mathbb{N}$ and $b, b' \in \mathbb{N}$ such that $\{a, a'\} \neq \{b, b'\}$." Why is the implication ("there exist at least two pairs ...") true?

   (c) In this section, the proof states,
   "If $a \neq a'$ and $b \neq b'$, then n has at least four factors, so $n \in S$.
   It may be true that $a = a'$ or $b = b'$. If $a = a'$, though, then n is a perfect square and $b \neq b'$, since there is only one positive square root possible for every n. Similarly, if $b = b'$, then $a \neq a'$. In either case, n has at least three factors (either a, b, b' or a, a', b), so $n \in S$."

      i. What is the negation of the statement "$a = a'$ or $b = b'$"?

      ii. Give an example of n such that "$a = a'$ or $b = b'$" is true.

      iii. Give an example of n such that "$a = a'$ or $b = b'$" is false.

      iv. Explain the meaning of the quoted passage using the examples you gave in (3(c)ii) and (3(c)iii).

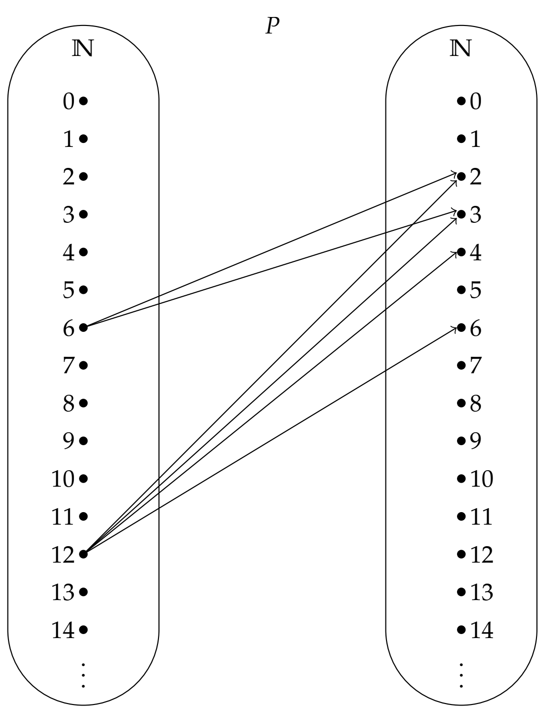
   (d) What is the logical reason for needing to show both that $S \subseteq T$ and $T \subseteq S$ to establish Proposition 1.5?

   (e) Why does the truth of the passage in 3b mean that that every $n \in T$ is also a member of S?

   (f) Walk through the steps of the entire proof of Proposition 1.5 using the example of $n = 12$.

   (g) Why does the first line of each part of the proof (why $S \subseteq T$ and why $T \subseteq S$) not apply to the case $n = 6$?

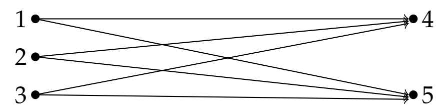
4. The diagram below shows P, a collection of arrows from a natural number to its parents. Some arrows below have been filled in. For example, P assigns 6 to 2 and 3, and assigns 12 to $2, 3, 4, 6$. Draw in three more arrows from a natural number to its parents.



5. Let D and R be sets. The **Cartesian product** of D and R, which we will work with more next time, is defined as the set of ordered pairs $\{(x, y) : x \in D, y \in R\}$.
   Example. If $D = \{1, 2, 3\}$ and $R = \{4, 5\}$, then $D \times R$ is the set

$$\{(1,4), (2,4), (3,4), (1,5), (2,5), (3,5)\}.$$

   One way to think about it is that it is all the pairs you can get by drawing all possible arrows from elements of D to elements of R when you draw the elements lined up in parallel to each other:



   Each arrow represents an ordered pair, with the starting point of the arrow being the first coordinate of the ordered pair and the ending point of the arrow being the second coordinate of the ordered pair.

   Your turn. Let $A = \{5, 6, 10\}$ and $B = \{-1, -2, -3\}$. Draw an arrow representation of the Cartesian product $A \times B$ and list its elements as ordered pairs.