Simulation of Practice Written Assignment: Explaining Power Properties

Students in your algebra class have defined rational and integer powers as below. Your goal for today is to move the class toward understanding different ways to interpret the meaning of $a^{-1/n}$.

Let a be a positive real number. Then we define a^{-1} and $a^{1/n}$ as follows.

Power	Definition
a^{-1}	$1/a$, the multiplicative inverse of a.
$a^{1/n}$	The positive n-th root of a. A number r is an n-th root of a if $r^n = a$.
$\sqrt[n]{a}$	Another way to write $a^{1/n}$

As students are working on simplifying expressions with rational powers, you observe the following work:

Student 1	Student 2
$27^{-1/3} = (27^{1/3})^{-1} = \frac{1}{27^{1/3}} = 27^3$	$27^{-1/3} = (27^{-1})^{1/3} = \left(\frac{1}{27}\right)^{1/3} = \sqrt[3]{\frac{1}{27}}$

For this simulation of practice, **write a mini-lesson plan** in which you clearly describe how you will conduct a whole class discussion which will allow you to elicit student thinking about these expressions. In your mini-lesson plan,

- Describe how the lesson might unfold toward the goal of understanding different ways to interpret $a^{-1/n}$, where a is positive and n is a nonzero natural number, as you:
  - Integrate both of the examples of student work,
  - Move the class discussion toward the goal
- Describe what you would say and do so that students can further investigate the ideas,
- Present a task or sequence of tasks you would give to students that serves the goal.

Your mini-lesson plan should indicate your understandings about power properties and power definitions and be no more than 2 pages in length. Your mini-lesson plan should be specific in describing what you do and say, and in the tasks that you would present.

Feedback Chart

Descriptor	Meets Expectations	Does Not Meet Expectations
Does the lesson plan integrate both of the examples of student work?	Both examples are integrated logically in a way that will move student thinking forward.	One or both examples are not logically integrated and/or will not move student thinking forward.
Is the whole class discussion plan reasonable?	Discussion questions are included that follow a logical path and will move student thinking forward.	Discussion questions are not logically sequenced and/or not appropriate to move thinking forward.
Are anticipated student responses for the whole class discussion reasonable?	Anticipated student responses are reasonable.	Anticipated student responses are unreasonable or not included.
Will the task posed move students towards understanding different ways to interpret $a^{-1/n}$?	Posed task will move students toward understanding different ways to interpret $a^{-1/n}$.	Posed task does not logically follow from previous discussion and/or will not move students toward understanding different ways to interpret $a^{-1/n}$.

Reflection Prompt (to be completed after receiving feedback):

1. What are some take-aways for you about using student thinking in moving toward a learning goal?
2. When you teach this concept in the future, what will you change? What will you keep? For what reasons?

17. (a) Solve for x and express your responses as an interval.

$$\log_{\frac{1}{2}} 3x < \log_2(3-2x)$$

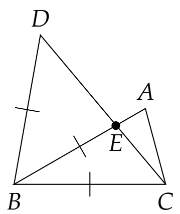
(b) Solve the question in a different way.

(c) Suppose you use these two solutions as example with your class. What would you want them to learn from these examples?

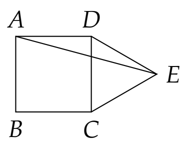
4 Homework 4

Working with Radians

1. ΔABC and ΔBCD are isosceles triangles, as marked.
   The measure of $\angle BAC$ is $75°$, the measure of $\angle BEC$ is $\frac{10}{18}\pi$.
   Find the absolute measure of angle D, in radians.
   Explain your reasoning.

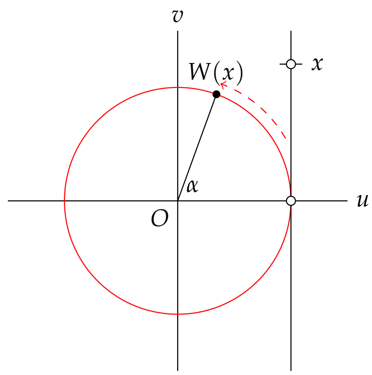


2. $ABCD$ is a square, and CDE is an equilateral triangle.
   What is the absolute measure of angle $\angle AED$?
   Explain your reasoning.



Warming up to the Wrapping Function

3. The length of an arc on a circle of radius 4 is θ radians. The endpoints of the arc are A and B, and we travel from A to B along the arc by moving counterclockwise. The center of the circle is O.
   (a) Draw a diagram of this situation.
   (b) What is the measure of the directed angle from $\overrightarrow{OA}$ to $\overrightarrow{OB}$? Explain your reasoning.
   (c) What is the measure of the directed angle from $\overrightarrow{OB}$ to $\overrightarrow{OA}$? Explain your reasoning.
4. The length of an arc on a circle of radius $1/4$ is θ radians. The endpoints of the arc are A and B, and we travel from A to B along the arc by moving counterclockwise. The center of the circle is O.
   (a) Draw a diagram of this situation.
   (b) What is the measure of the directed angle from $\overrightarrow{OA}$ to $\overrightarrow{OB}$? Explain your reasoning.
   (c) What is the measure of the directed angle from $\overrightarrow{OB}$ to $\overrightarrow{OA}$? Explain your reasoning.
5. The length of an arc on a unit circle is θ radians. The endpoints of the arc are A and B, and we travel from A to B along the arc by moving counterclockwise. The center of the circle is O.
   (a) Draw a diagram of this situation.
   (b) What is the measure of the directed angle from $\overrightarrow{OA}$ to $\overrightarrow{OB}$? Explain your reasoning.
   (c) What is the measure of the directed angle from $\overrightarrow{OB}$ to $\overrightarrow{OA}$? Explain your reasoning.
6. Let W denote the wrapping function as defined in our notes. If $\alpha = 70°$, and find the length of the segment between the two open circles. Explain your reasoning.



Wrapping Function for Sine and Cosine

7. Explain why $\sin^2 x + \cos^2 x = 1$, using the wrapping function definition of sine and cosine, and the Pythagorean Theorem.
8. Order from smallest to largest:
$$\sin(0.1), \sin(2.1), \sin(3.1), \sin(6.1), \sin(6.5)$$
9. Order from smallest to largest:
$$\cos(0.1), \cos(2.1), \cos(3.1), \cos(6.1), \cos(6.5)$$
10. Express $\cos(x - \frac{\pi}{2})$ in terms of sine.
    (a) Draw a wrapping diagram to illustrate your identity.
    (b) Prove your identity using the wrapping function definition of sine and cosine.
11. Express $\sin(x - \frac{\pi}{2})$ in terms of cosine.
    (a) Draw a wrapping diagram to illustrate your identity.
    (b) Prove your identity using the wrapping function definition of sine and cosine.
12. Derive an expression for each of the following using the identities already established (see the end of Lesson 4 notes). Justify your work using a 2-column proof format.
    (a) $\cos(x + y)$
    (b) $\sin(x + y)$
    (c) $\sin(x - y)$