Simulation of Practice: Concept of Inverse

Suppose that you are teaching high school pre-calculus and you are introducing the concept of inverse of an invertible function. The students have already learned the definition of invertible function. Here is a task that you plan to use:

Opening Task: Water in the Pot

A pot has straight vertical sides, stands 6 inches tall, and has 2 gallon capacity. Water is being poured into this pot.

(a) Draw a picture of this pot.

(b) Find the relationship between v, for a volume of water poured into the pot, and h, the height of water in the pot at that volume. Graph this relationship.

(c) If there is $\frac{3}{4}$ gallon of water in the pot, how high is the water level?

(d) If the water level in the pot is 3 inches high, how much water is in the pot?

(e) If the water level is 3.5 inches high, how much water is in the pot?

You would like your class to understand both of the following definitions, and how these definitions can be interpreted using different representations, especially algebraic, graphical, and verbal.

Definition 1. Given an invertible function f, the inverse of f is the function that maps $y \mapsto x$ whenever $x \mapsto y$ is an assignment of f. The inverse function is denoted f^{-1}.

Definition 2. Given an invertible function f, the inverse of f is the function such that for all x in the domain of f, we have $f^{-1} \circ f(x) = x$.

Break down your plan into the following sections. In communicating this plan, place these sections separately from each other and label them. The page limit for this plan is 3 pages, typed (3 pages single-sided, or 1 page double-sided plus 1 page single-sided).

- **Goals for the lesson**. Write this to be consistent with the scenario described above as well as the sections below. You may need to revise/reword this part as you work through the rest of the planning. (This is a very normal thing to happen when planning a lesson.)
- **Solution to the Opening Task**. Include three different solutions to (e). Two of these should be mathematically correct. One should be mathematically incorrect while showing reasoning you think it is likely that a student would do.
- **Immediately following the Opening Task.** Write down what you would say after the class has completed the activity to introduce the concept of inverse of a function. In this description, work in the notation you would use to refer the function being inverted, how you would define this function and its inverse, and how these have to do with the Opening Task.
- **Key terminology.** State the order that you would bring in each of Definitions 1 and 2. Then list the phrases or terminology in these definitions that you think students would benefit from discussion to understand.
- **Illustrating concept with multiple representations.** Describe how you would use the problem context, algebraic notation, and graph to help students make sense of these phrases or terminology, and then Definition 2.
  - What are 3 questions you would ask to lead a discussion on this?
  - For each question, what would you anticipate students to respond?
  - For each question, what would an ideal response be?
- **Mathematical equivalence of definitions.** Suppose you ask the class the question: “Why are these two definitions saying the same thing?”
  - What would you anticipate students to respond to this question?
  - Describe two ideal responses, one using the example, and one that is a general explanation.
- **Summary.** Write down a summary sentence or two that you would use to conclude the discussion. This sentences should be short enough to be easy to say, and drive home the main mathematical point of the lesson.
- **Follow up.** Write a task you would assign for homework to launch a discussion on how to understand the following statement: if f is an invertible function and $y = f(x)$, then $x = f^{-1}(y)$.

Feedback Chart

Descriptor	**Meets Expectations**	**Does Not Meet Expectations**
Are anticipated solutions realistic?	Both correct and incorrect solutions are generated using a variety of methods.	Solutions are not reasonable and/or do not include a variety of method and/or no correct solution is provided.
Is the whole class discussion plan reasonable?	Discussion questions are included that follow a logical path and will move student thinking forward.	Discussion questions are not logically sequenced and/or not appropriate to move thinking forward.
Are anticipated student responses for the whole class discussion reasonable?	Anticipated student responses are reasonable.	Anticipated student responses are unreasonable or not included.
Will the task posed move thinking toward understanding the statement, *if f is an invertible function and $y = f(x)$, then $x = f^{-1}(y)$*?	Task provided will move students toward understanding statement.	Task provided does not logically follow from previous and/or will not move toward understanding the statement.

Reflection Prompt (to be completed after receiving feedback)

1. What are some take-aways for you about using student thinking in moving toward a learning goal?
2. When you teach this concept in the future, what will you change? What will you keep? For what reasons?