## 3 In-Class Resources for Lesson 3

## The Power of One (and Zero)

Find all pairs of $a, b$ such that $a^{b}=1$.

Find all pairs of $a, b$ such that $a^{b}=0$.

## FUN WITH POWERS

Find the solutions.

1. $\frac{5^{x^{2}}}{5^{4 x} 5^{2}}=5^{3}$
2. $2^{\left(a^{2}-9\right)}=1$
3. $3^{\left(4^{\left(5^{x}-4\right)}-3\right)}=3$.
4. $a^{a\left(a^{2}-4\right)}=1$.

## ARE ALL NUMBERS POWERS OF A GIVEN NUMBER?

1. Using the calculator here, https://www.desmos.com/scientific, estimate $x$ such that $5^{x}=74$, so that you are within the first two decimal places of the exact value of $x$.
2. How do you know that the first 2 decimal digits are correct?
3. Let $P$ be a random positive real number. Is it always possible to find an $x \in \mathbb{R}$ such that $5^{x}=P$ ? Why or why not?

Takeaway:

## Finding the Exponent

Here are two approximations:

$$
5^{1.37}=9, \quad 5^{0.43}=2 .
$$

Using these values, we can make other approximations. For example:

$$
\begin{array}{ccl}
{\text { Power } 5^{x}}^{x} & \text { Exponent } x & \\
5^{[\square}=81 & \boxed{?}=2.74 & 81 \\
& =9^{2} & \text { Reasoning } \\
& =5^{1.37} \cdot 5^{1.37} \quad \text { (given) } \\
& & =5^{2.74} \quad \text { (product of power property) } \\
& \boxed{?} & =2.74
\end{array}
$$

Using the information below, and without a calculator, complete the following table.

| Power $5^{x}$ | Exponent $x$ |  |
| :---: | :---: | :---: |
| $5^{x}=8$ |  |  |
| $5^{x}=18$ |  |  |
| $5^{x}=45$ |  |  |

Play with the numbers in the table below to complete the table.
You can use what we have already done and these ideas:
$5^{1.37}=9, \quad 5^{0.43}=2, \quad 5^{2.74}=81$

| Power $5^{x}$ | Exponent $x$ |  |
| :---: | :---: | :---: |
| $5^{x}=10$ |  |  |
| $5^{x}=100$ |  |  |
| $5^{x}=900$ |  |  |

## Exponential functions: Invertible?

Let $f(x)=5^{x}$. Fill in sample entries of an input/output table to represent the inverse of $f$.


Is $f(x)$ invertible?
Is any exponential function an invertible function?

## Generalizing how to find logs

Let $L=\log _{5} 9$ and $M=\log _{5} 2$.

1. Complete the given rows as shown.
2. Then find as many other powers of 5 as you can that you can write in terms of $L$ and $M$. Record your findings in the table.
\(\left.$$
\begin{array}{c|c|c}\text { Power } P & \begin{array}{c}\text { Representation in terms of known } \\
\text { powers of 5 }\end{array} & \begin{array}{c}\text { Use the representation to express } \\
\log _{5} P\end{array}
$$ <br>
\hline \hline 9 \& - \& \log _{5} 9=L <br>

\hline 2 \& - \& \log _{5} 2=M\end{array}\right]\)| $\log _{5} 9^{2}=2 L$ |
| :--- |
| 81 |
| $\frac{9^{3}}{5^{20}}$ |

