

1. Prove by induction on  $n$  that

$$n! > 4n^2$$

for all integers  $n \geq 5$ .

2. Let  $f_n$  denote the  $n$ th Fibonacci number ( $f_0 = 0$ ,  $f_1 = 1$ , and  $f_{n+2} = f_{n+1} + f_n$ ). Prove by induction on  $n$  that

$$\sum_{i=1}^n f_{2i} = f_{2n+1} - 1$$

for all positive integers  $n$ .