Math 3323 | **MATRIX ALGEBRA** | Fall 2009

Dr. Duval | **Investigation 5**

due Friday, November 13

How to Use the "Linear Transformations" Module

This module is designed to display the effect of a linear transformation on $\mathbf{R}^2$.

The matrix is formed by dragging the points B' and D', which show the effect of the linear transformation on points from the square B and D respectively; the left column of this matrix is (the coordinates of) D', and the right column of this matrix is (the coordinates of) B'.

The point E' shows the effect of this linear transformation (multiplication by the matrix) on the vector (from the origin to the point) E. You may drag the point E freely, and E' will move accordingly.

- The Reset button changes the matrix to the identity matrix (yellow square). It does not change the point E, though it necessarily moves E' to E. (Why?)

- The Show Grid button displays a grid of points, which allows you to place points more accurately. The Hide Grid button turns this option off.

- The Hide Figures button hides the yellow square and the purple parallelogram; this lets you focus on the relation between E and E' for a fixed matrix. The Show Figures button restores these figures.

Assignment

1. Recall that a matrix is singular when the corresponding parallelogram is "flat" (has no interior). Pick several singular matrices. In each case, change the position of the point E. What can be said about its image E'? What can be said about the determinant?

2. Pick a matrix with a positive determinant. (You may have to play around to see how to do this.) Rotate the point E around the origin and watch the behavior of its image E'. Do the same for a matrix with negative determinant. What can you conjecture about how E' moves in each of these cases? Try several examples of each kind of matrix to test your conjecture, and report on the results.

3. For a **wide variety** of matrices, do the following. Change the position of the point E and watch the image E'. Try to find such places of E in the plane such that

   the points A, E, and E' all lie on one line.

   (a) Is it possible to do this for all matrices? Can you categorize the types of matrices for which it is possible, and the types for which it is not?

   (b) For each matrix where you can do this at all, in how many directions can you place E so that all three points lie on the same line? How are these directions related, if at all? Do your answers depend on the type of matrix you picked? If so, how?

   (c) In the case where all three points are collinear, move the point E along the line in concern and watch the ratio of the vector lengths: $\overrightarrow{AE'}/\overrightarrow{AE}$. What do you notice about this ratio? Does your answer depend on the type of matrix you picked? If so, how?

   (d) Did you notice anything else interesting about this problem of placing all three points on the same line?