Math 3323 | **MATRIX ALGEBRA** | Fall 2009
Dr. Duval | **Investigation 3**
due Friday, October 16

How to Use the "Vectors – Single Linear Combination" Module

This module is designed to display vectors and vector operations from a geometrical perspective. The module is divided into three sections, each providing different tools. This assignment will just use the sections Vectors and Construction of Single Linear Combination.

Under the Vectors section, vectors are associated with the names i, j, k, and l. [Here, i, j, and k do **not** represent the unit coordinate vectors.] Any vector can be entered including the stored vectors labeled $a0$ – $a14$. The vector $a0$ is the zero vector. Each vector is represented by a thick line segment with a color code. The module runs by Mathematica software. In Mathematica, vectors are entered using curly set notation. For instance, Mathematica recognizes $\{1, 2, 3\}$ as a vector. The red dot is the origin (the point $\{0, 0, 0\}$).

Under the Construction of Single Linear Combination section, you can assign values for the symbols d and e. These values produce thin lines which are parallel copies (with the same colors as the vectors represented) of scalar multiples of the pair of vectors k and l. For instance, when the values $d = 2$ and $e = 3$ are entered, the program produces thin lines with the same color as the vectors k and l, providing the length of the lines as twice the first vector and three times the second vector respectively. This section also has a window to enter new vectors under the name w.

General graphing commands: Hitting the Graph button will graph whatever values you have specified. Once the graph is made, pressing the Shift button (on your keyboard) and dragging the mouse will resize the graph. Moving the mouse will rotate the graph.

Assignment

Before each question, make sure to reset each vector to the zero vector namely a0 and each value to 0.

1. First we graph some vectors and some linear combinations.
   (a) Enter the vectors $a1$ and $a2$ for k and l respectively. Then, enter the value 1 for both d and e. Now, enter the vector, $a1 + a2$, for vector i. Make sure the j vector has the zero vector, $a0$, assigned. Click Graph. What do you observe?
   (b) Repeat the experiment in question 1a with the following new examples:
      i. $d = 1$, $e = 2$, $i = a1 + 2a2$.
      ii. $d = 2$, $e = 2$, $i = 2a1 + 2a2$.
      iii. $d = 1$, $e = -1$, $i = a1 - a2$.
      iv. $d = -1$, $e = -1$, $i = -a1 - a2$.
   (c) Enter your own set of values (several examples) for d and e, and next enter $da1 + ea2$ (for each value of d and e) vector for i. State your observation for each set of values of d and e. Remember to enter vectors $a1$ and $a2$ for k and l respectively making sure the j vector has the zero vector, $a0$, assigned.

2. Now we try to find linear combinations to match a given vector, if possible.

   First, reset all values to 0 and vectors to a0.

   Enter the vector $a1$ for k and the vector $a2$ for l. Enter each vector below for w and search the values (if they exist) of d and e (testing out different values for d and e by entering them into the module) that give $w = d(a1) + e(a2)$. Record the values of d and e for each vector that satisfy $w = d(a1) + e(a2)$. If there are no such values, explain why not.

   (a) $w = 4, 8, 2$.
   (b) $w = 0, 0, 1$.
   (c) $w = 1, 2, 1/2$.
   (d) $w = 1, 1, 1$.

3. Finally, analyze your observations in the previous questions.
   (a) Based on what you obtained and observed on questions 1 and 2, describe the characteristics of the vectors $d(a1) + e(a2)$ for all real number values of d and e. Explain your answer carefully.
   (b) What geometric figure would you observe if one considers the collection of all the vectors of the form $d(a1) + e(a2)$ for all real number values of d and e? Explain.