Math 3323 | **MATRIX ALGEBRA** | Fall 2009
Dr. Duval | **Investigation 2**
due Friday, October 2

How to Use the "Matrix Product" Module

This module is designed to display matrices and their products from a geometric perspective.

The blue matrix is formed by dragging the points P and Q; the left column of this matrix is (the coordinates of) P, and the right column of this matrix is (the coordinates of) Q. Similarly, the purple matrix is formed by dragging the points R and S; the left column of this matrix is (the coordinates of) R, and the right column of this matrix is (the coordinates of) S.

The product of the blue matrix times the purple matrix is displayed as the gray matrix, and the product of the purple matrix times the blue matrix is displayed as the brown matrix. The columns of each of these matrices give the coordinates of some of the corner points of the corresponding parallelogram.

- The Reset button changes both matrices to the identity matrix (yellow square).
- The Show Grid button displays a grid of points, which allows you to place points more accurately. The Hide Grid button turns this option off.

Assignment

1. Describe as carefully as you can how the blue and purple parallelograms are formed from the points P, Q, R, and S.

2. Notice that the gray and brown matrices are generally not the same. What property of matrix multiplication does this reflect? (Perhaps it's more accurate to ask what property matrix multiplication **doesn't** have that causes this to happen.)

3. Recall that an **inverse** of a matrix A is a new matrix B such that AB is the identity matrix. (The identity matrix in this setup is the yellow square.)

   (a) Drag P and Q to new positions in the diagram, picking a new blue matrix. Now figure out where to drag R and S to make the purple matrix the inverse of the blue matrix, if possible. In other words, move the purple parallelogram so that the resulting gray parallelogram exactly (or as close to exactly as you can manage) covers the yellow square. Verify that the gray matrix is (at least close to) the identity matrix. What do you notice about the brown matrix?

   (b) Repeat the process in step 3a several more times, and record your observations.

   (c) Are you able to find a blue matrix for which you **cannot** find an inverse?

4. Recall that a matrix is **singular** if its columns are linearly dependent. Since we are working with matrices with only two columns, this means the columns are linearly dependent precisely when one is a multiple of the other. This means, for instance, that the blue matrix is singular when P and Q lie on the same line, making the blue parallelogram "flat", *i.e.*, have no interior.

   (a) Make the blue parallelogram "flat", by putting points P and Q on the same line. What happens to the gray and brown product matrices? Try changing the purple parallelogram; does this change your observation in this question? Explain your answers.

   (b) Reset the diagram, and repeat step 4a above with the purple parallelogram: Make it "flat", i.e., have no interior, by putting points R and S on the same line (or as close as you can manage). Once again, observe what happens to the gray and brown product matrices. State your observations.

   (c) Can you find other ways to get the brown and/or gray product matrices to be flat? If so, explain what they are.

5. Revisit question 3 after completing question 4. Did your work on question 4 make you change your answers to any parts of question 3? Why or why not?