Sinusoidal Signals: Amplitude Modulation (AM)

Scott A. Starks, PhD, PE
Professor of Electrical & Computer Engineering

Amplitude Modulation

- **Amplitude Modulation** (**AM**) is a technique used in electronic communication, most commonly for transmitting information via a radio carrier waveform.

General Idea Behind AM

- AM works by varying the strength (amplitude) of the carrier in proportion to a modulation (message) waveform that is being send from a source to a destination.
- The modulation waveform may, for instance, correspond to the sounds to be reproduced by a loudspeaker or the light intensity of television pixels.

Carrier Waveform

- A carrier waveform, $c(t)$, is just a sinusoid of frequency f_c, with an amplitude A, and a phase angle of 0.
- It may be expressed mathematically as

$$c(t) = A\cos(2\pi\ f_c\ t)$$

- Note: The frequency of the carrier waveform is typically a high value. For commercial AM, f_c, is in the range of 560 – 1,720 KHz.

Modulation (Message) Waveform

- Let $m(t)$ represent the modulation waveform.
- The modulation waveform, $m(t)$, represents the message that we wish to transmit from one point (source) to another (destination.)
- We place a restriction on m(t), namely that

  $|m(t)| < 1$ for all values of t.

An AM Waveform is Produced by Multiplication

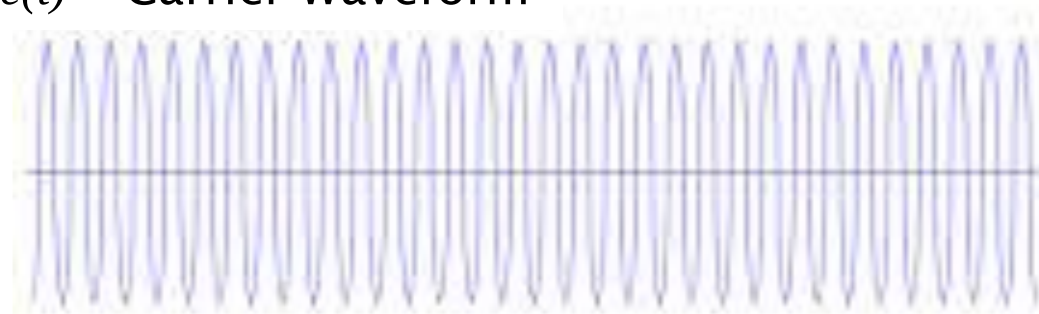
- Amplitude modulation results when the carrier $c(t)$ is multiplied by the positive quantity $(1+m(t))$

$$y(t) = (1 + m(t))\, c(t)$$

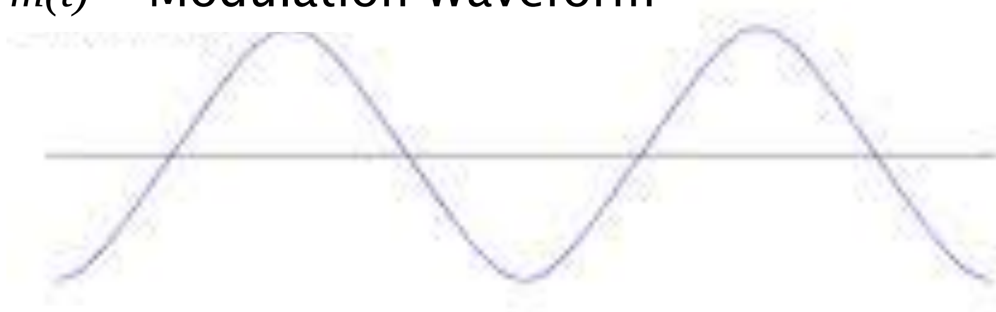
$$y(t) = (1 + m(t))\, (A\cos(2\pi\, f_c t))$$

- So, $y(t)$ is the resulting AM signal.
- The process used to create the AM waveform is called **AM Modulation**.

c(t) – Carrier Waveform



m(t) – Modulation Waveform



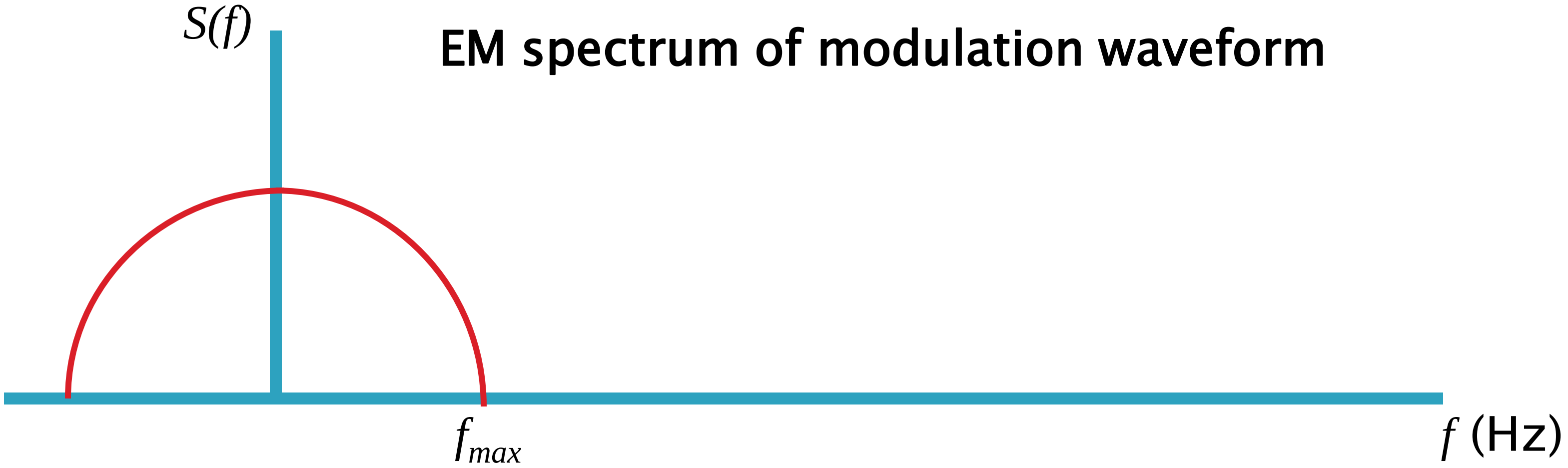
y(t) – AM Waveform

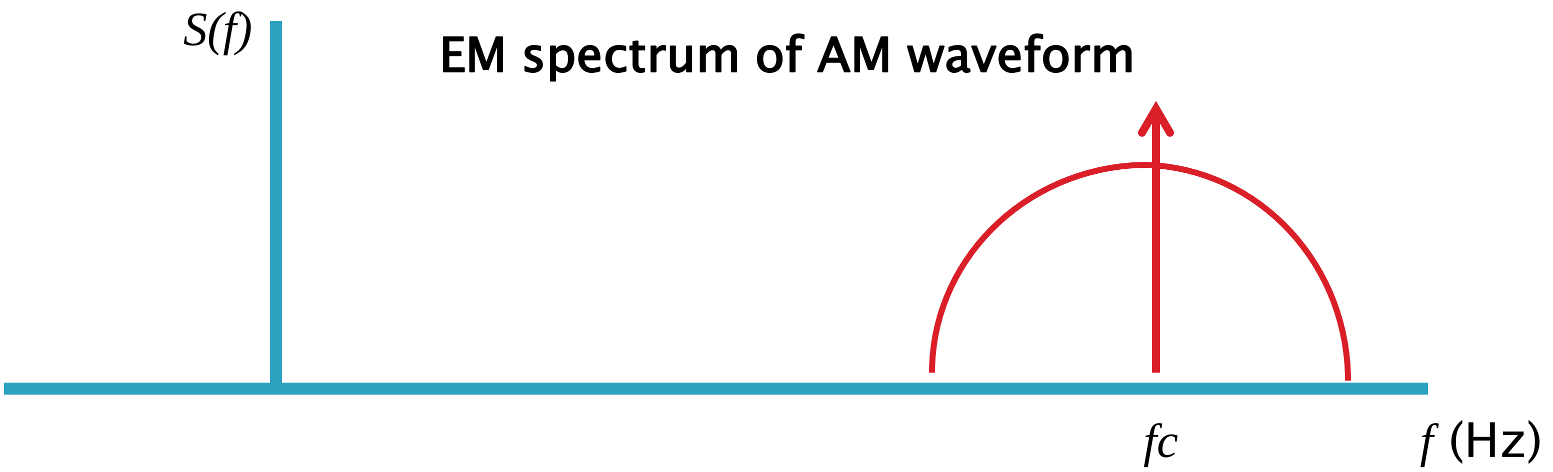
from www.technologyuk.net

Observation

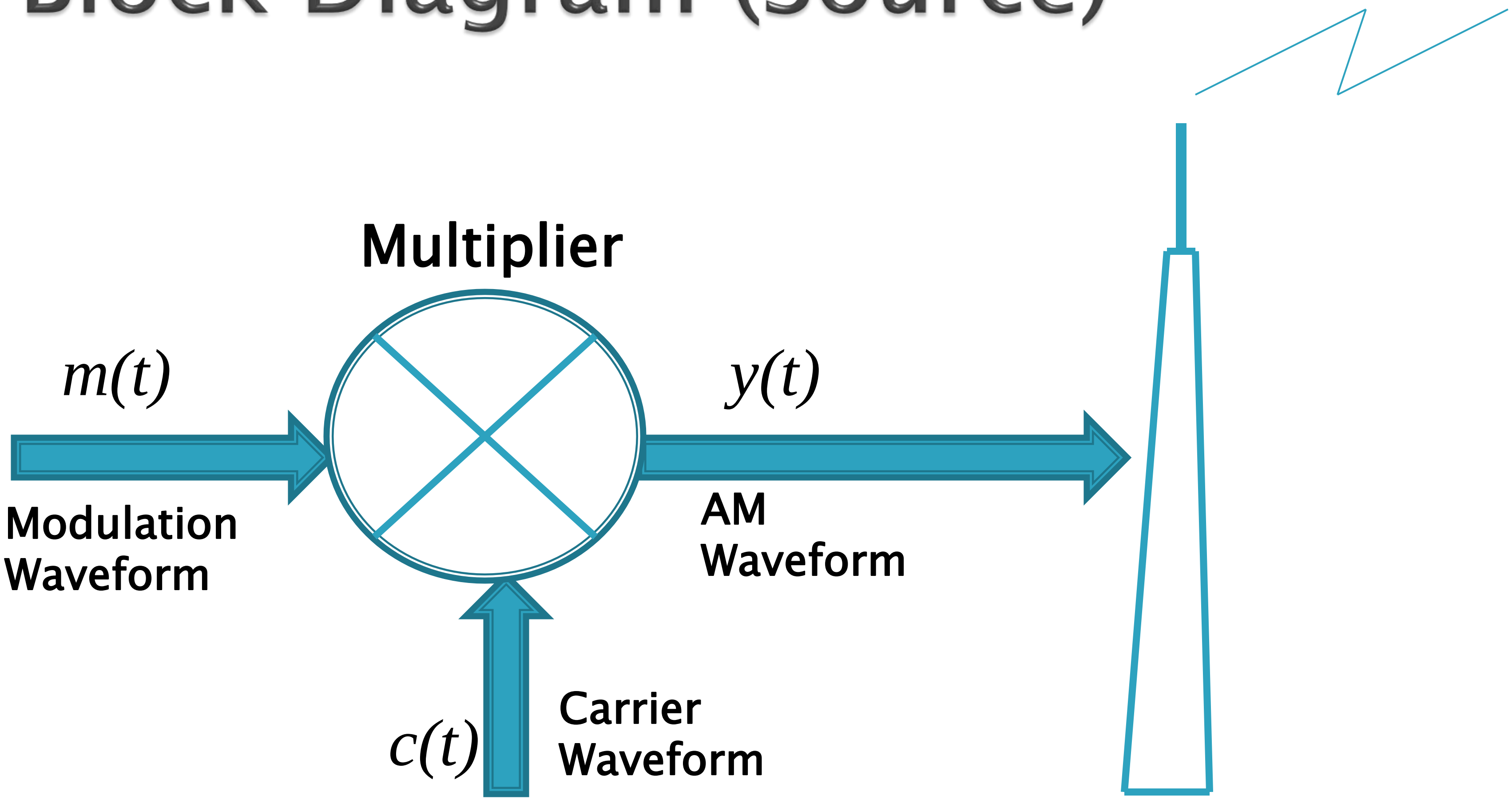
- Multiplying a signal of relatively low frequency, such as a modulation waveform, by a very high frequency carrier waveform, has the effect of shifting the electromagnetic energy of the product waveform up in frequency to a region of the electromagnetic spectrum centered at the frequency of the carrier waveform.

Electromagnetic Spectrum





Block Diagram (Source)



Broadcast Antenna: Transmits the AM Waveform through free space.

AM Reception

- At the destination, an Antenna receives the AM signal.
- The received AM signal, $y(t)$, will then be processed (**demodulated**) in such a way as to recover the original modulation signal, $m(t)$.
- This process is called **Demodulation**.

Demodulation

- First, the received AM waveform is multiplied by a sinusoidal waveform having the same frequency and phase as the original carrier.
- The result of this multiplication is called the product waveform, $p(t)$.
- For the sake of discussion, we assume the amplitude of the product waveform to be 1.

$$p(t) = y(t)\cos(2\pi f_c t)$$

Analysis of the Product Waveform

AM waveform

Sinusoid of Equal Frequency & Phase as Carrier

$$p(t) = y(t)\cos(2\pi f_c t)$$

$$p(t) = ((1 + m(t))\cos(2\pi f_c t))\cos(2\pi f_c t)$$

AM waveform

$$p(t) = (1 + m(t))\cos^2(2\pi f_c t)$$

Important Trig Formula

- We recall an important formula from Trigonometry:

$$\cos^2(\alpha) = \frac{1}{2} + \frac{1}{2}\cos(2\alpha)$$

AM Demodulation

- We apply the trig formula to the expression for the product waveform:

$$p(t) = (1 + m(t))\cos^2(2\pi\ f_c t)$$

$$= (1 + m(t))\left[\frac{1}{2} + \frac{1}{2}\cos(2\pi(2f_c)t)\right]$$

$$= \frac{(1 + m(t))}{2} + \frac{(1 + m(t))\cos(2\pi(2f_c)t)}{2}$$

AM Demodulation (cont.)

$$p(t) = \frac{1}{2} + \frac{m(t)}{2} + \boxed{\frac{\left((1+m(t)\right)\cos(2\pi(2f_c)t)}{2}}$$

- The third term represents an additional AM signal with a carrier frequency of 2 f_c, which is very high.
- This component of $p(t)$ can be removed by means of a low pass filter. Nothing more than an RC circuit similar to the one that we previously discussed.

AM Demodulation (cont.)

$$p_1(t) = \frac{1}{2} + \frac{m(t)}{2}$$

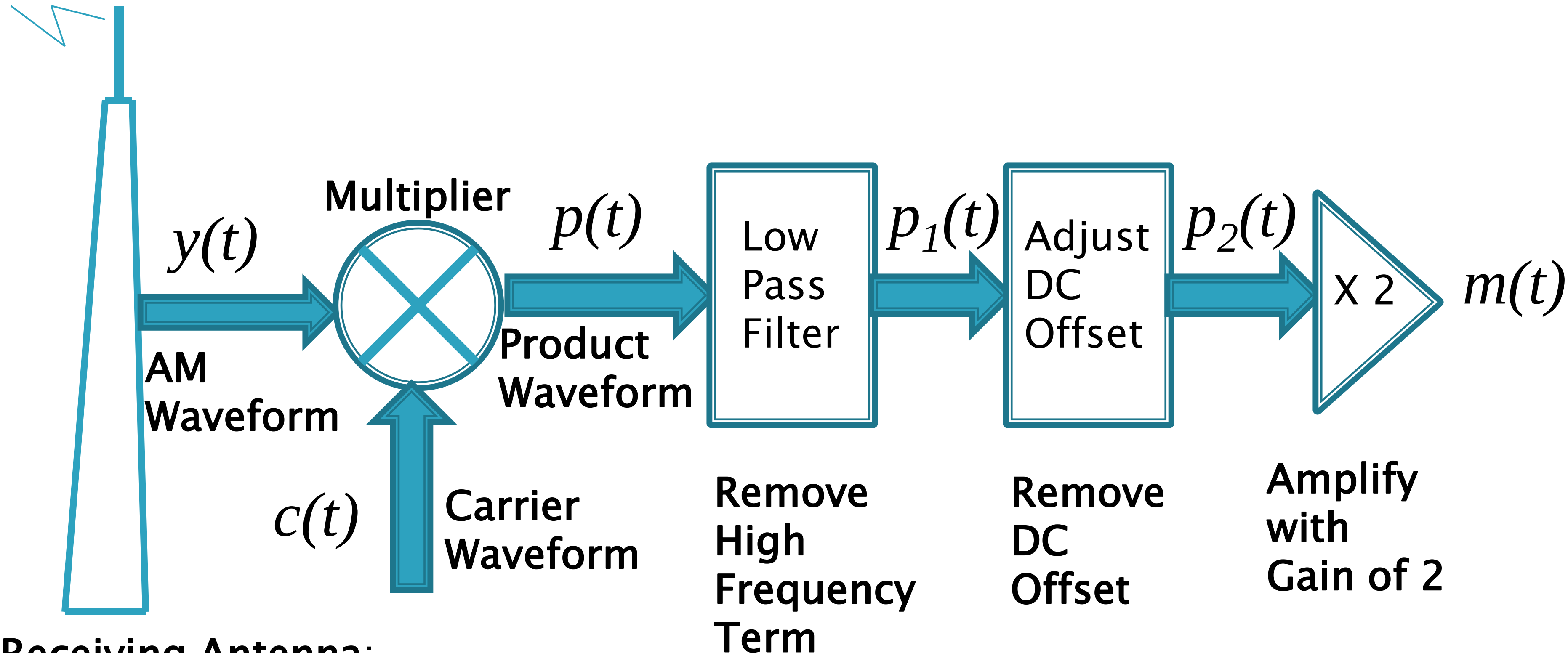
- The first remaining term of p(t) is a constant. This is what engineers call a “DC offset.” A DC offset can be removed by changing the physical location of what is known as “ground” in electric circuitry. This term is easily removed by just measuring voltages with respect to a different reference.

AM Demodulation (cont.)

$$p_2(t) = \frac{m(t)}{2}$$

- That leaves us with one remaining term. This term is a scaled version of the original modulation signal. We can pass this term through an amplifier with a gain of 2. That will allow us to recover an exact copy of the original modulation signal, m(t).

Block Diagram (Destination)



Receiving Antenna: Receives the AM Waveform from free space.

The original modulation signal, m(t), is recovered exactly!

Summary

- Trigonometry plays a crucial role in Amplitude Modulation schemes.
- AM signals are merely high frequency sinusoids with varying amplitudes.
- A trigonometric identity is used in the analysis of an AM processing scheme.