Week 14 Math 1508 Review for Exam 3

1. Evaluate the six trigonometric functions at the real number
   a. $t = \frac{5\pi}{6}$

$$\sin\left(\frac{5\pi}{6}\right) = \frac{1}{2}, \quad \cos\left(\frac{5\pi}{6}\right) = -\frac{\sqrt{3}}{2}, \quad \tan\left(\frac{5\pi}{6}\right) = -\frac{\sqrt{3}}{3}$$

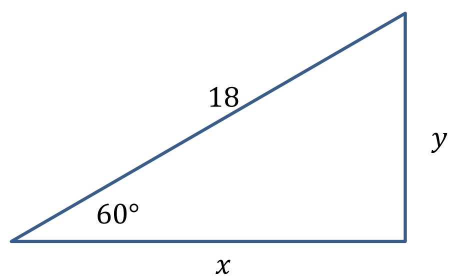
$$\csc\left(\frac{5\pi}{6}\right) = 2, \quad \sec\left(\frac{5\pi}{6}\right) = -\frac{2\sqrt{3}}{3}, \quad \cot\left(\frac{5\pi}{6}\right) = -\sqrt{3}$$

   b. $t = -\pi$

$$\sin(-\pi) = 0, \quad \cos(-\pi) = -1, \quad \tan(-\pi) = 0$$

$$\csc(-\pi) = undefined, \quad \sec(-\pi) = -1, \quad \cot(-\pi) = undefined$$

2. Find the exact values of x and y for the following.
   a.



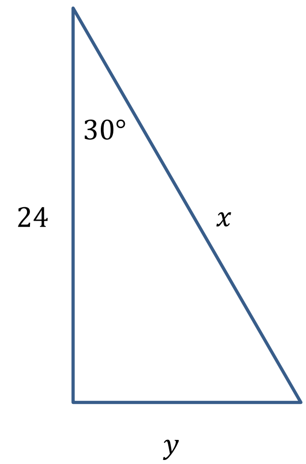
$$\sin(60°) = \frac{y}{18}$$

$$9\sqrt{3} = y$$

$$\cos(60°) = \frac{x}{18}$$

$$9 = x$$

b.



$$x = 16\sqrt{3}, \qquad y = 8\sqrt{3}$$

3. Find the exact values of the six trigonometric functions of θ with the given constraint.

   a. $\cos\theta = -\frac{4}{5}$ *with constraint* $\tan\theta < 0$

$$\sin(\theta) = \frac{3}{5}, \quad \cos(\theta) = -\frac{4}{5}, \qquad \tan(\theta) = -\frac{3}{4}$$

$$\csc(\theta) = \frac{5}{3}, \quad \sec(\theta) = -\frac{5}{4}, \qquad \cot(\theta) = -\frac{4}{3}$$

   b. $\sec\theta = -2$ *with constraint* $\sin\theta < 0$

$$\sin(\theta) = -\frac{\sqrt{3}}{2}, \quad \cos(\theta) = -\frac{1}{2}, \qquad \tan(\theta) = \sqrt{3}$$

$$\csc(\theta) = -\frac{2\sqrt{3}}{3}, \quad \sec(\theta) = -2, \qquad \cot(\theta) = \frac{\sqrt{3}}{3}$$

   c. $\cot\theta$ *is undefined given that* $\frac{\pi}{2} \leq \theta \leq \frac{3\pi}{2}$

$$\sin(\theta) = 0, \quad \cos(\theta) = -1, \qquad \tan(\theta) = 0$$

$$\csc(\theta) = undefined, \quad \sec(\theta) = -1, \qquad \cot(\theta) = undefined$$

4. Graph the following functions. Be sure to include 1) the amplitude; 2) its period; 3) Five key points in one period.

   a. $y = -1\cos\left(\frac{x}{4} - \frac{\pi}{4}\right) - 3$

$$amplitude: a = |-1| = 1$$

The period is: $\frac{2\pi}{b}$

$$period = \frac{2\pi}{\left(\frac{1}{4}\right)} = 8\pi$$

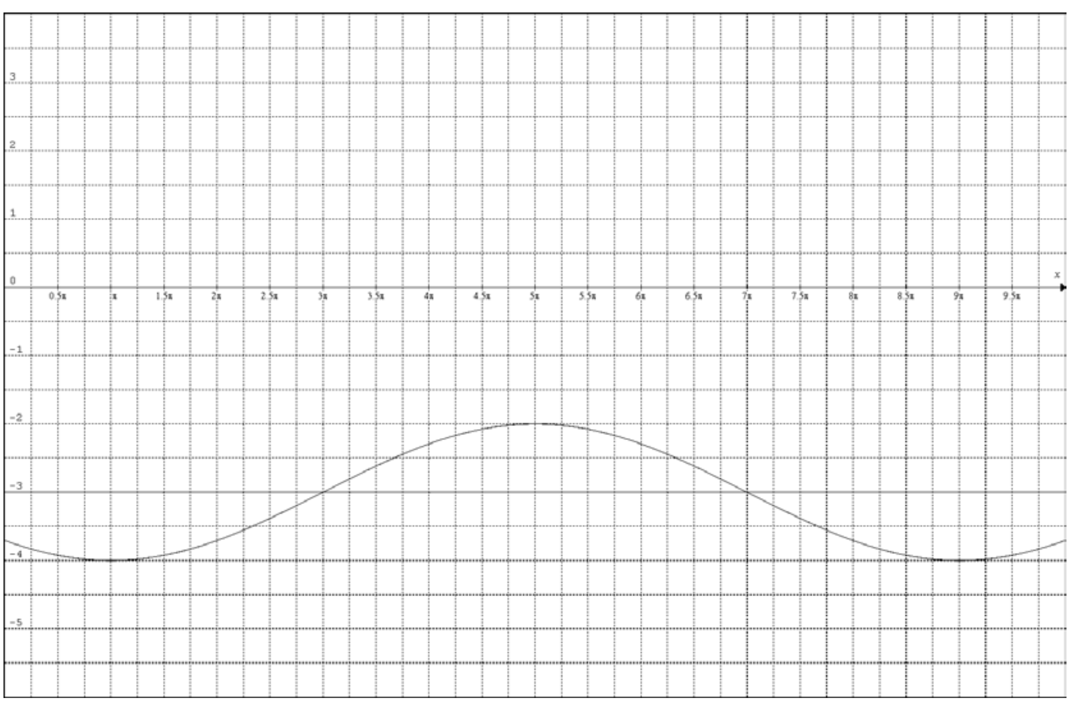
Five key points: Solve the following five equations

$$\frac{x}{4} - \frac{\pi}{4} = 0, \quad \frac{x}{4} - \frac{\pi}{4} = \frac{\pi}{2}, \quad \frac{x}{4} - \frac{\pi}{4} = \pi, \quad \frac{x}{4} - \frac{\pi}{4} = \frac{3\pi}{2}, \quad \frac{x}{4} - \frac{\pi}{4} = 2\pi$$

$$x = \pi, \quad x = 3\pi, \quad x = 5\pi, \quad x = 7\pi, \quad x = 9\pi$$

Five key points:

$\underset{(\pi,-4)}{minimum}$, $\underset{(3\pi,-3)}{midline\ intercept}$, $\underset{(5\pi,-2)}{maximum}$, $\underset{(7\pi,-3)}{midline\ intercept}$, $\underset{(9\pi,-4)}{minimum}$



b. $y = \csc(x - \pi)$

$$amplitude: a = |1| = 1$$

The period is: $\frac{2\pi}{b}$

$$period = \frac{2\pi}{1} = 2\pi$$

Five key points: Solve the following five equations

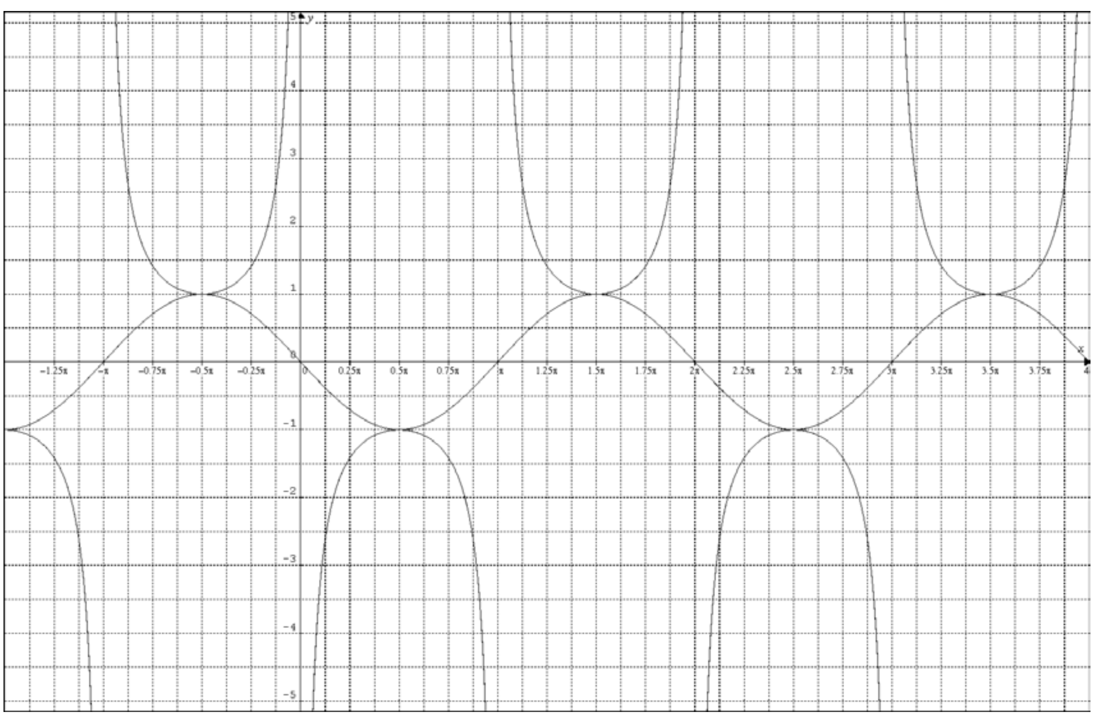
$$x - \pi = 0, \quad x - \pi = \frac{\pi}{2}, \quad x - \pi = \pi, \quad x - \pi = \frac{3\pi}{2}, \quad x - \pi = 2\pi$$

$$x = \pi, \quad x = \frac{3\pi}{2}, \quad x = 2\pi, \quad x = \frac{5\pi}{2}, \quad x = 3\pi$$

Minimum and Maximum points:

$$\underset{minimum}{\left(\frac{3\pi}{2}, 1\right)}, \quad \underset{maximum}{\left(\frac{5\pi}{2}, -1\right)}, \quad \underset{minimum}{\left(\frac{7\pi}{2}, 1\right)}$$

Vertical Asymptotes: $x = -\pi$, $x = \pi$, $x = 2\pi$ and $x = 3\pi$



5. Find the exact value of the expression (draw a right triangle)
   a. $\csc\left[\arctan\left(-\frac{5}{12}\right)\right]$

$$\csc\left[\arctan\left(-\frac{5}{12}\right)\right] = -\frac{13}{5}$$

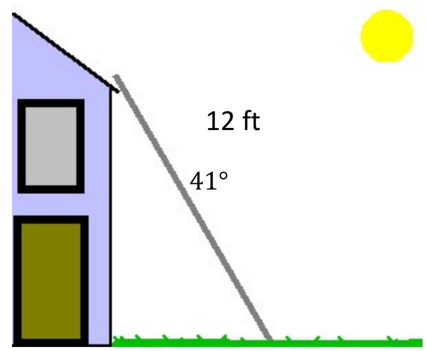
   b. $\tan\left[\arcsin\left(-\frac{3}{4}\right)\right]$

$$\tan\left[\arcsin\left(-\frac{3}{4}\right)\right] = -\frac{3\sqrt{7}}{7}$$

   c. $\csc\left[\arccos\left(-\frac{\sqrt{3}}{2}\right)\right]$

$$\csc\left[\arccos\left(-\frac{\sqrt{3}}{2}\right)\right] = 2$$

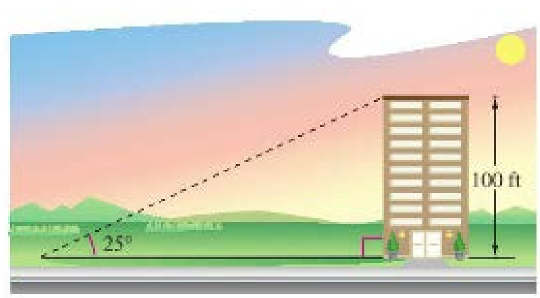
6. It's time to turn on your evaporative cooler in your house. You place a 12 foot ladder on your house to get to the roof. The angle of elevation is 41°, how tall is the house? Round your answer to two decimal places.



$$hieght = 7.87\ ft$$

7. The sun is 25° above the horizon. Find the length of a shadow cast by a building that is 100 feet tall. Round your answer to three decimal places.

$$length\ of\ shadow = 214.45\ ft$$



8. Solve the equation for the interval $[0,2\pi]$

a. $\csc(x) - 2 = 0$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}$$

b. $2\cos^2(x) + \cos(x) - 1 = 0$

$$x = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$$

c. $\cos(x) + \sin(x)\tan(x) = 2$

$$x = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$$

d. $2\sin(2x) + \sqrt{3} = 0$

$$x = \frac{2\pi}{3}, \frac{5\pi}{6}$$

9. Verify the identity

a. $\cos x + \sin x \tan x = \sec x$

$$\cos x + \sin x \, tan\ x = \cos x + \sin x \left(\frac{\sin x}{\cos x}\right)$$

$$= \cos x + \frac{\sin^2 x}{\cos x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos x}$$

$$= \frac{1}{\cos x} \text{ Since } (\cos^2 x + \sin^2 x = 1)$$

$$= \sec x$$

b. $\frac{1}{\sin x} - \frac{1}{\csc x} = \csc x - \sin x$

$\frac{1}{\sin x} - \frac{1}{\csc x} = \csc x - \sin x$

Since $\csc x = \frac{1}{\sin x}$, $\sin x = \frac{1}{\csc x}$

c. $\frac{1}{\cos x+1} + \frac{1}{\cos x-1} = -2\csc x \, cot\, x$

Find a common denominator

$$\frac{1}{cos\, x+1} + \frac{1}{cos\, x-1}$$

$$= \frac{\cos x - 1 + \cos x + 1}{cos^2 x - 1}$$

$$= \frac{2\cos x}{-\sin^2 x}$$

$$= -2\ \frac{1}{\sin x}\ \frac{\cos x}{\sin x}$$

$$= -2\csc x cot\, x$$

d. $\cos(\pi - \theta) + \sin\left(\frac{\pi}{2} + \theta\right) = 0$

$$cos(\pi - \theta) + \sin\left(\frac{\pi}{2} + \theta\right)$$

$$= (\cos\pi\cos\theta + \sin\pi\sin\theta) + \left(\sin\frac{\pi}{2} cos\theta + \cos\frac{\pi}{2}\sin\theta\right)$$

$$= [(-1)\cos\theta + (0)\sin\theta] + [(1)\cos\theta + (0)\sin\theta]$$

$$= -\cos\theta + \cos\theta = 0$$

10. Use the Law of Sines (if two exists, find both) or the Law of Cosines to solve the triangle. Round your answer to two decimal places.

a. $B = 72°, C = 82°, b = 54$

Since we have AAS case, use the law of sines:

$$c = \frac{54 \sin 82^{\circ}}{\sin 72^{\circ}} = 56.23$$

$$a = \frac{54\ \sin 26^{\circ}}{\sin 72°} = 24.89$$

$$A = 180^{\circ} - (72^{\circ} + 82^{\circ}) = 26^{\circ}$$

b. $A = 75°, a = 51.2, b = 33.7$

We have SSA case which is an ambiguous case. First find angle B using the law of sines

$$B = \sin^{-1}(.636) = 39.48^{\circ}$$

Since a second B would be $B_2 = 140.3^{\circ}$ which cannot be used since $140.3^{\circ} + 75^{\circ} = 215.3^{\circ} > 180^{\circ}$, **we have only one triangle** and $B = 39.48^{\circ}$

$$C = 180^{\circ} - (39.48^{\circ} + 75^{\circ}) = 65.52^{\circ}$$

$$c = \frac{51.2 \sin 65.52^{\circ}}{\sin 75^{\circ}} = 48.24$$

c. $a = 6, b = 9, c = 14$

We have an SSS case so we use the law of cosine.

$$C = \cos^{-1}(-.73) = 137.01^{\circ}$$

Now we have SSA and we can use the law of sines to find angle B:

$$B = \sin^{-1}(.439) = 25.99^{\circ}$$

$$A = 180 - (137.01^{\circ} + 25.99^{\circ}) = 16.99^{\circ}$$