Investigation
Linear Transformations
Math3323

Name:
Spring 2009

For this homework use “Matrix” module at the site
http://www.math.utep.edu:8080/jsp/index.html (you may need to copy and paste the link).

Module Description

The module is designed to teach the connection between a 2x2 matrix and a linear transformation of the plane.

A yellow unit square ABCD is displayed in the Cartesian plane. The matrix displayed in the left upper corner is the transformation matrix defined by identifying the images of the two unit vectors AD and AB. The user changes the matrix by dragging the points D' and B' to any place in the plane. The resulting purple parallelogram AB'C'D' is the image of the yellow square under the defined transformation.

Once the matrix is defined, the image of any vector can be obtained. This can be observed in the vector AE' which is the image of the vector AE. As the point E is dragged along the plane, the image point E' changes its position accordingly. The square and the parallelogram can be turned off by the “Hide Figures” button; and can be turned back on by the “Show Figures” button.

If the feature “Turn Traces On” is selected, one can create an arbitrary picture on the plane by moving the point E, and watch its image made by the point E'. The traces can be turned off by the “Turn Traces Off” button, and can be erased by clicking on a red “X” button in the right bottom corner of the window.

Additional displays on this module are: the determinant of the matrix, the ratio of the distance AE'/AE and the coordinates of the point E.

The “Reset” button sets back the matrix to the identity matrix.

The “Show/Hide Grid” buttons put/hide the integer grid on the plane;
Assignment

1. Click the "Hide Figures" button. First we try to figure out which locations of E will send E' to the origin.

2. What happens to E' when we move E to the origin?

3. Try to find other locations of E that send E' to the origin? Are there any? If so, where are they? Explain your answer.

Next we try to figure out what are the possible locations we can send E' to by picking E appropriately.

4. Click the "Turn Traces On" button.

5. Drag the point E around; notice how much of the grid is covered by the point E' (these are indicated by blue dots). Do you think you could cover the whole grid? Are there any points you can't reach with blue dots? Explain your answer.

Now we repeat steps 1-5 above with different matrices. Click the “Show Figures” button.

6. Reset the diagram, and then move the purple parallelogram to a new location by dragging B' and D' to a new location. Make sure the purple parallelogram still has an interior (make sure B' and D' are not on the same line). Repeat steps 1-5 above. Do this for several different choices of purple parallelograms.

7. What happens if we make the purple parallelogram "flat" (singular), i.e., have no interior? Put B' and D' on the same line (or as close as you can manage). Repeat steps 1-5 above. Do you notice any difference in your answers? Explain your answer. Also, describe as carefully as you can in this case:

   7a. What are the locations of E that send E' to the origin? In other words, what region is traced out by the green dots (locations of E) as you try to keep E' at the origin?

   7b. What are the possible locations of E'? In other words, what region is traced out by the blue dots?