1.7 Transformations of Functions

Now that we know the basic functions, we can move them around the coordinate plane. The way we do this is through vertical and horizontal shifts, reflections, and stretches.

Vertical and Horizontal Shifts – Let *c* be a positive real number. Vertical and horizontal shifts in the graph of $y = f(x)$ are represented as follows.

1. Vertical shifts *c* units upward: $h(x) = f(x) + c$
2. Vertical shifts *c* units downward: $h(x) = f(x) - c$

outside is vertical

3. Horizontal shift *c* units to the right: $h(x) = f(x - c)$
4. Horizontal shift *c* units to the left: $h(x) = f(x + c)$

inside is horizontal

Reflections in the Coordinate Axes – Reflections in the coordinate axes of the graph of *y = f(x)* are represented as follows.

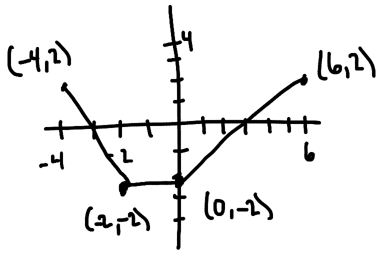
1. Reflections in the x-axis: $h(x) = -f(x)$ vertical reflection uses x-axis as mirror
2. Reflections in the y-axis: $h(x) = f(-x)$ horizontal reflection uses y-axis as mirror

Shifts and reflections are called rigid transformations because they keep the shape of the graph rigid as they move. Non-rigid transformations include stretching and shrinking graphs; transformations that cause a distortion in the graph.

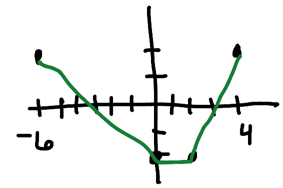
Vertical distortions: For $y = f(x)$, the transformation given by $g(x) = cf(x)$ is a vertical stretch if $c > 1$ and a vertical shrink if $0 < c < 1$.

Horizontal distortions: For $y = f(x)$, the transformation given by $g(x) = f(cx)$ is a horizontal shrink if $c > 1$ and a horizontal stretch is $0 < c < 1$.

Examples: Use the given graph to sketch the indicated transformations.



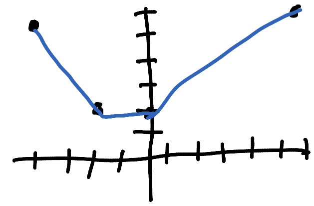
① $y = f(-x)$



horizontal reflection across y-axis.

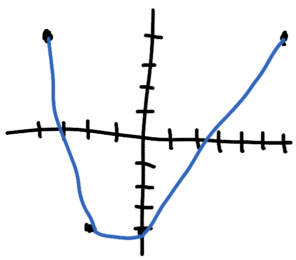
② $y = f(x) + 4$

vertical shift up 4



(-4,6)
(-2,2)
(0,2)
(6,6)

③ $y = 2f(x)$

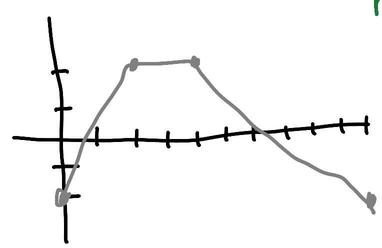


multiply y values by 2

(-4,4) (-2,-4)
(0,-4) (6,4)

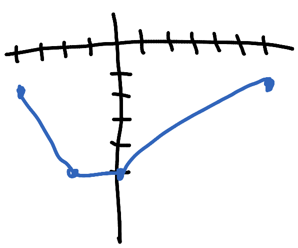
④ $y = -f(x-4)$

vert reflection
right 4 units



(0,-2)
(2,2)
(4,2)
(10,-2)

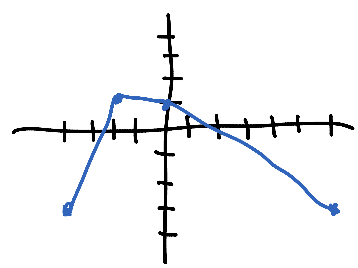
⑤ $y = f(x) - 3$ down 3



(-4,-1)
(-2,-5)
(0,-5)
(6,-1)

⑥ $y = -f(x) - 1$

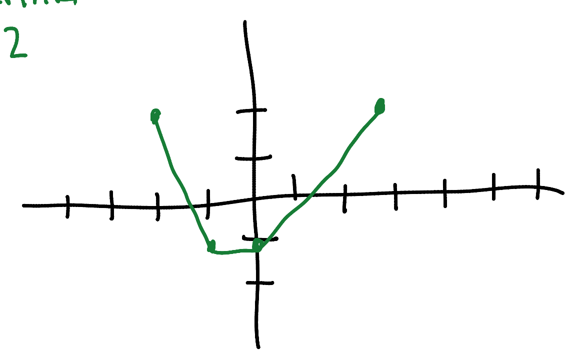
vertical reflection then down 1



(-4,-3)
(-2,1)
(0,1)
(6,-3)

⑦ $y = f(2x)$ horizontal shrink by a factor of 2

(-2,2) (-1,-2) (0,-2) (3,2)

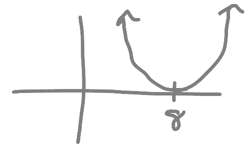


Examples: The given function is related to one of the parent functions described in 1.6. (a) Identify the parent function *f*. (b) Describe the sequence of transformations from *f* to *g*. (c) Sketch the graph of *g*. (d) Use function notation to write *g* in terms of *f*.

1. $g(x)=(x-8)^2$

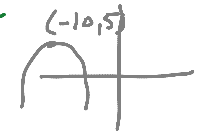
$f(x)=x^2$ shifted right 8 units

$g(x)=f(x-8)$



2. $g(x)=-(x+10)^2+5$

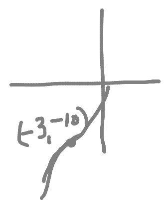
$f(x)=x^2$ reflect (vert) across x-axis, left 10, up 5



$g(x)=-f(x+10)+5$

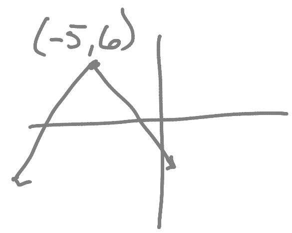
3. $g(x)=(x+3)^3-10$

$f(x)=x^3$ left 3, down 10



$g(x)=f(x+3)-10$

4. $g(x)=6-|x+5|$



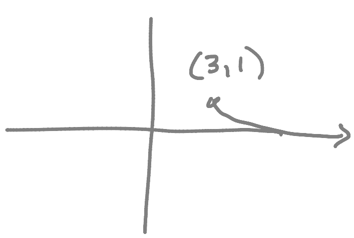
$f(x)=|x|$ up 6, reflect (vert) across x-axis, left 5

$g(x)=6-f(x+5)$

5. $g(x)=-\frac{1}{2}\sqrt{x-3}+1$

$f(x)=\sqrt{x}$

reflect (vert) across x-axis, vert shrink $\frac{1}{2}$, right 3, up 1



$g(x)=-\frac{1}{2}f(x-3)+1$

Examples: Write an equation for the function that is described by the given characteristics.

1. The shape of the quadratic function, but shifted three units to the right and seven units downward.

$$f(x) = (x-3)^2 - 7 \quad \text{or} \quad f(x) = (x-3)^2 - 7$$

2. The shape of the absolute value function, but shifted four units to the left and eight units upward.

$$f(x) = |x+4| + 8 \quad \text{or} \quad f(x) = |x+4| + 8$$

3. The shape of the radical function, but shifted nine units upward and reflected in both the x-axis and y-axis.

$$f(x) = -\sqrt{-x} + 9 \quad \text{or} \quad f(x) = -\sqrt{-x} + 9$$