7.2 Cardinality

**Definition:** If $A$ is a finite set, then its cardinality is $n(A) = \text{number of elements in } A$.

**Union Rule of Counting:** If $A$ and $B$ are finite sets, then $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

**Definition:** If $S$ is a finite universal set and $A$ is a subset of $S$, then $n(A') = n(S) - n(A)$ and $n(A) = n(S) - n(A')$.

**Definition:** If $A_n$ and $B$ are finite sets, then $n(A \times B) = n(A)n(B)$.

Examples: Let $A = \{\text{Dirk, Johan, Frans, Sarie}\}$, $B = \{\text{Frans, Sarie, Tina, Klaas, Henrika}\}$, and $C = \{\text{Hans, Frans}\}$. Find the numbers indicated.

1. $n(A) + n(B) = 4 + 5 = 9$
   - 4 elements in $A$
   - 5 elements in $B$

2. $n(A \cup B) = 7$
   - $A \cup B = \{\text{Dirk, Johan, Frans, Sarie, Tina, Klaas, Henrika}\}$

3. $n(A \cup (B \cap C)) = 4$
   - $B \cap C = \{\text{Frans}\}$
   - $A \cup (B \cap C) = \{\text{Dirk, Johan, Frans, Sarie}\}$
Example: Let \( C = \{ \text{Head, Tail} \} \), \( D = \{ 1, 2, 3, 4, 5, 6 \} \), and \( P = \{ \text{red, yellow, blue} \} \). Find the numbers indicated.

1. \( n(C \times C) = 2 \times 2 = 4 \)
2. \( n(D \times D) = 6 \times 6 = 36 \)
3. \( n(C \times P) = 2 \times 3 = 6 \)
4. \( n(C \times D \times P) = 2 \times 6 \times 3 = 36 \)