

## PDE2D GUI Example 2

Example 2 is a nonlinear 2D steady-state problem

$$\begin{aligned}U_{xx} + U_{yy} &= V \\V_{xx} + V_{yy} &= 1 + 0.2U^2\end{aligned}$$

in an hour glass shaped region. This is a fourth order elastic plate problem, with nonlinear loading, but reduced here to a system of two second order equations. On the left and right parabolic boundaries, there are clamped boundary conditions,  $U = dU/dn = 0$  and on the top and bottom there are simply supported conditions,  $U = V = 0$ .

The parametric equations used are:

$$\begin{aligned}X &= P1 * (1 + P2^2) \\Y &= P2\end{aligned}$$

where  $-1 < P1 < 1$ ,  $-1 < P2 < 1$ .

## Welcome to the PDE2D 9.4 GUI

Continue

The PDE2D GUI can be used to access the PDE2D collocation methods, which use cubic finite elements. These methods can handle 0D and 1D problems, and 2D and 3D problems in "a wide range of simple regions." For problems in complex 2D regions you should use the PDE2D Galerkin method, which is accessible only through the PDE2D Interactive Driver (type "pde2d [progname]").

If you want to see one of three prepared examples, select the example number below. If you ask to see an example, do not enter any data, just press the "Continue" buttons and look at the prepared input.

Show example number  
(0 if none)

\*\*\*\*Example 1. A 3D eigenvalue problem

$$Q_{xx} + Q_{yy} + Q_{zz} - Q/\sqrt{x^2+y^2+z^2} = \lambda Q$$

in half of a torus, with  $Q=0$  on the curved surface of the torus and on one flat end, and  $dQ/dn+Q=0$  ( $dQ/dn$ =normal derivative) on the other flat end. We will look for the eigenvalue closest to -1.15.

\*\*\*\*Example 2. A 2D nonlinear steady-state problem

$$U_{xx}+U_{yy} = V$$

$$V_{xx}+V_{yy} = 1 + b*U^2$$

in an hour-glass shaped region. This is a fourth order elastic plate problem, with nonlinear loading, but reduced here to a system of two second order equations. On the left and right parabolic boundaries there are clamped boundary conditions,  $U=dU/dn=0$ , and on the flat top and bottom there are simply supported conditions,  $U=V=0$ .

\*\*\*\*Example 3. A 1D time-dependent problem

$$U_t = V$$

$$V_t = -A*V + C^2*U_{xx}$$

with initial conditions  $U(x,0) = \max(0,1-|x-5|)$ ,  $V(x,0)=0$  and boundary conditions  $U_x=V_x=0$  at  $x=0$  and  $x=10$ . This is the damped wave equation  $U_{tt} + A*U_t = C^2*U_{xx}$ , reduced to a system of two equations by defining  $V=U_t$ .

Continue

## Begin Describing your PDE Problem

Dimension

Problem type

Number of PDEs (NEQN)

Precision  (Double strongly recommended)

Linear?  (If unsure, assume nonlinear)

Continue

## Fortran expressions

You will be asked to input "Fortran expressions" to define your PDE coefficients, boundary conditions (BC) and other parameters. Fortran expressions are almost the same as MATLAB expressions, which use +, -, \*, / to represent the four arithmetic operators, and recognize functions such as sin,cos,log,exp,sqrt,abs,max... One important difference is that A to the B power is represented as A\*\*B rather than A^B. Note also that Fortran is not case-sensitive.

These expressions may reference the independent variables T,X,Y,Z,P1,P2,P3 (depending on the dimension of the problem). Some may also reference the unknowns and their first and second spatial (not time) derivatives; these are written using the names you assign, for example if you name an unknown "PHI", refer to it and its x-derivatives as PHI,PHIx,PHIxx. Your expressions may all reference the "parameters" you define, and also the parameter "pi" (3.14159...).

Expressions which cannot be written on a single line may reference Fortran functions, which you may define in a separate file. You will be prompted for the file name at the end of this session.

Once you have clicked "Continue" on a screen, you cannot return to correct or modify your expressions. However, you will find it easy to make such changes in the Fortran program created by the PDE2D GUI, which is well-documented. There you will also be able to modify the default values set by the GUI for numerous other options, or add further Fortran code.

Define parameters (constants)

Parameter names are 1-6 alphanumeric characters beginning with a letter. Names beginning with I,J,K,L,M,N must be integers. Parameters are constants, not functions of T,X,... Parameter definitions may reference previously-defined parameters.

Continue

Parameter Name

Value (Constant or Fortran expression, up to 65 characters)

<input type="text" value="b"/>	=	<input type="text" value="0.2 ! (comment) b is coefficient of nonlinear term"/>
<input type="text"/>	=	<input type="text"/>
<input type="text"/>	=	<input type="text"/>
<input type="text"/>	=	<input type="text"/>
<input type="text"/>	=	<input type="text"/>
<input type="text"/>	=	<input type="text"/>
<input type="text"/>	=	<input type="text"/>
<input type="text"/>	=	<input type="text"/>
<input type="text"/>	=	<input type="text"/>
<input type="text"/>	=	<input type="text"/>
<input type="text"/>	=	<input type="text"/>

Continue

## Define region using parametric equations

You can solve PDEs in a region using this GUI only if you can describe it as the set of points

$$(X(p_1, p_2), Y(p_1, p_2))$$

where  $A_1 < P_1 < B_1$ ,  $A_2 < P_2 < B_2$ , that is, if you can parameterize it (smoothly) using parameters with constant limits. For example, for a rectangle simply take  $X=P_1$ ,  $Y=P_2$ , and for a disk take  $X=P_1 \cos(P_2)$ ,  $Y=P_1 \sin(P_2)$ , where  $P_1, P_2$  are polar coordinates.

Define  $X(p_1, p_2)$ ,  $Y(p_1, p_2)$  below. Then if you assign, for example, a name "U" to an unknown, in future Fortran expressions reference its derivatives as  $U_x, U_y, U_{xx}, U_{yy}, U_{xy}$  ( $U_x = dU/dX$ , etc.). You may also reference  $X, Y, P_1, P_2$  and the derivatives of U with respect to the parameters as  $U_1, U_2, U_{11}, U_{22}, U_{12}$  ( $U_1 = dU/dP_1$ , etc.). In the boundary conditions you can also reference the normal derivative as  $U_{norm}$ .

Fortran expressions, up to 65 characters


X

Y

Define P1 and P2 grids.  $A1 < P1 < B1$ ,  $A2 < P2 < B2$ .

Continue


Default grid is uniform. A non-uniform grid can be specified in the Fortran program.

Number of P1-grid points (NP1GRID)  

Fortran expressions, up to 65 characters

A1

B1

Number of P2-grid points (NP2GRID)  

A2

B2

## 2D steady-state problem

Continue

$$F_1 = 0$$

$$F_2 = 0$$

:

where the  $F_i$  are functions of  $X, Y, P_1, P_2$  and the unknowns and their first and second derivatives.

Boundary conditions at  $P_1=A_1, B_1$  and  $P_2=A_2, B_2$  are:

$$G_1 = 0$$

$$G_2 = 0$$

:

where the  $G_i$  are functions of  $X, Y, P_1, P_2$  and the unknowns and their first derivatives; they may also reference the components  $NORM_x, NORM_y$  of the unit outward normal.



Name the unknowns

Names are 1-3 alphanumeric characters each, beginning with a letter A-H or O-Z.

U\_1 name

Use these names in all future input expressions.

For example, if U\_1 is named "U", then refer to this unknown and its derivatives as U,Ux,Uxx... in future Fortran expressions.

U\_2 name

U\_3 name

U\_4 name

U\_5 name

U\_6 name


U\_7 name

U\_8 name

## Integrals

Define any functions whose integrals you want to compute. They may be functions of the unknowns and their first and second derivatives, plus X,Y,P1,P2,T.

Continue

Number of integrals desired (NINT)  

Fortran expressions, up to 65 characters

Integral 1

Integral 2

Integral 3

Integral 4

Integral 5

Integral 6

Integral 7

Integral 8

Continue

Fortran expressions, up to 65 characters

F1

F2

F3

F4

F5

F6

F7

F8

Continue

Initial Guesses

Fortran expressions, up to 65 characters

U\_10=U0

U\_20=V0

U\_30

U\_40

U\_50

U\_60

U\_70

U\_80

Periodic BC?

For "no" BC, enter: None.

Continue

Define BCs at P1=A1

Fortran expressions, up to 65 characters

G1 U ! At left side, U=Unorm=0

G2 Unorm

G3

G4

G5

G6

G7

G8

Define BCs at P1=B1

G1 U ! At right side, U=Unorm=0

G2 Unorm

G3

G4

G5

G6

G7

G8

Periodic BC?

For "no" BC, enter: None.

Continue

Define BCs at P2=A2

Fortran expressions, up to 65 characters

G1 U ! At bottom, U=V=0

G2 V

G3

G4

G5

G6

G7

G8

Define BCs at P2=B2

G1 U ! At top, U=V=0

G2 V

G3

G4

G5

G6

G7

G8

## Fortran function file

If you referenced your own Fortran functions in any of your Fortran expressions, you may now supply the name of a file where these functions are defined (or will be when RUNPDE2D is executed). Remember to type functions and their arguments as "double precision" if they are double precision in the main program, and fixed-format must be used, so start lines in column 7 or later.

File name (if any)

Example Fortran function. The function below could be called (it would be referenced simply as "GB(P2,U,Unorm)") to specify boundary conditions on the perimeter ( $P1=B1$ ) of a disk, if polar coordinates ( $X=p1*\cos(p2)$ ,  $Y=p1*\sin(p2)$ ) are used for a 2D problem:

```

      function GB(P2,U,Unorm)
      double precision GB,P2,U,Unorm,pi
C234567 (parameters not available within functions)
      pi = 3.141592654d0
      if (P2 .le. pi/2) then
C          for P2 < pi/2, specify BC U=1
          GB = U-1
      else
C          for P2 > pi/2, specify BC Unorm=0
          GB = Unorm
      endif
      return
      end

```

Finish

## You have finished defining your PDE problem

The PDE2D Interactive Driver will now be run using as input the "pde2d.in" file created by the PDE2D GUI, and a Fortran program will be created, which can be executed using RUNPDE2D. This program contains comments which make it easy for you to make minor corrections and modifications to your model--the input you provided during the GUI session will be clearly marked by comments "INPUT FROM GUI". There you will also be able to select many options not documented in the GUI, for example, it will be easy to modify this program to request nonuniform grids, to compute all eigenvalues of an eigenvalue problem, to compute boundary integrals of functions of the solution and its derivatives, or to add off-diagonal terms to your "C" matrix (for time-dependent problems) or "RHO" matrix (for eigenvalue problems).

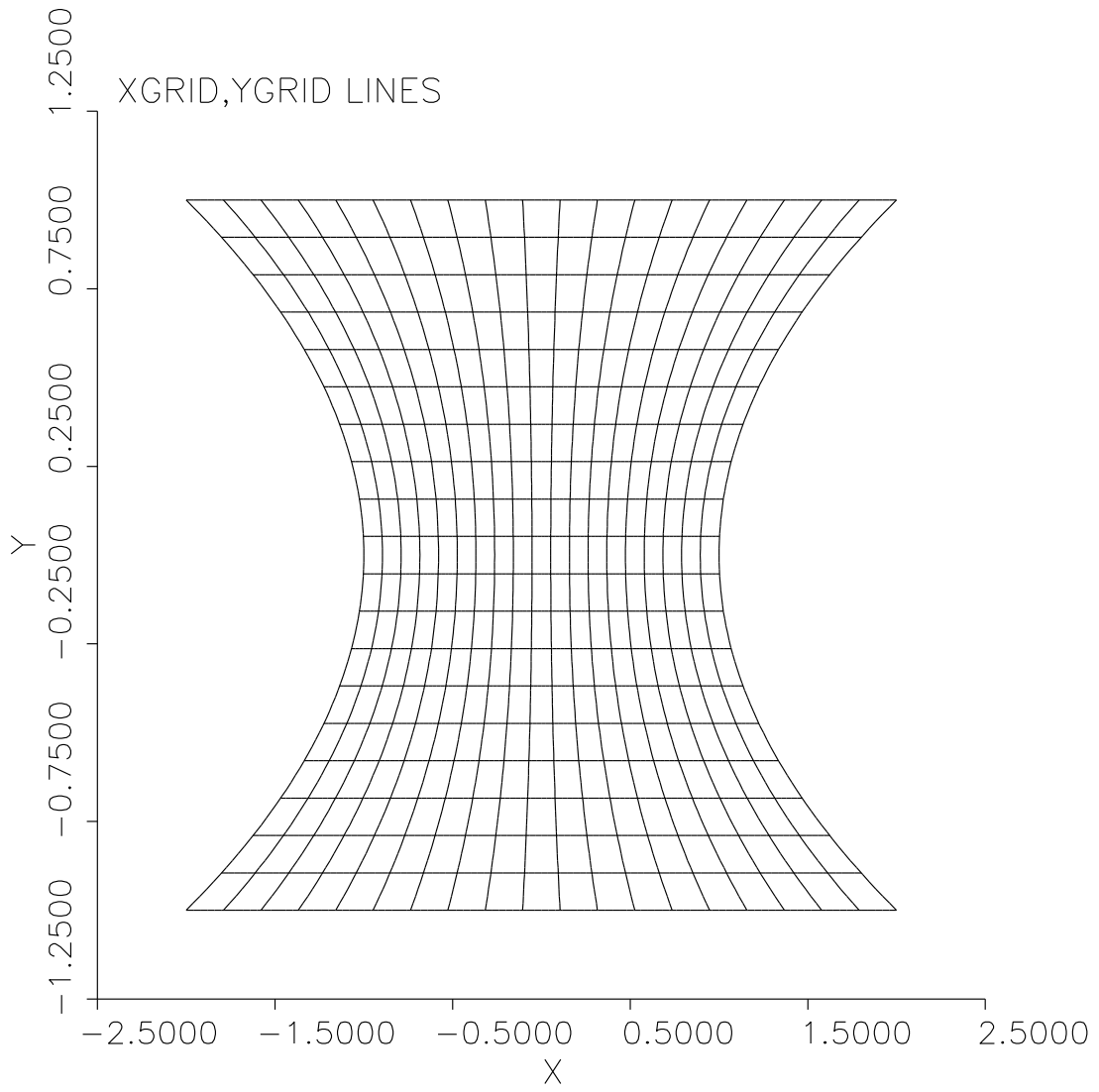
You do NOT need to be a Fortran programmer to make these modifications, and you do not need to work through another GUI session unless you have to change one of the options on the second GUI page. However, if you want to solve problems in complex 2D regions, or if you have more than eight PDEs, you will need to construct your program using the PDE2D Interactive Driver.

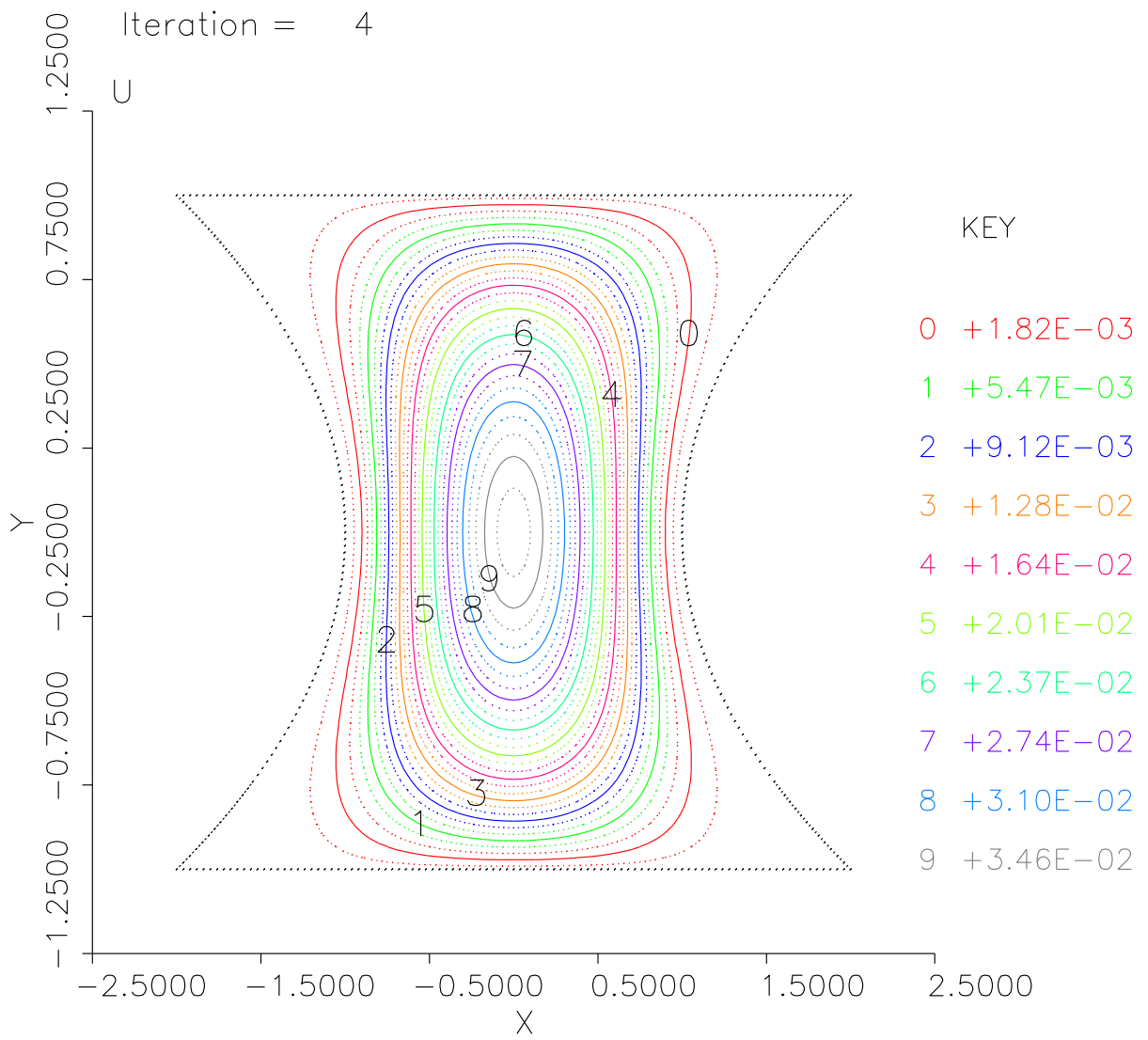
When you execute RUNPDE2D, plots of all unknowns will be made, but you can easily modify the program to postprocess the solution in many other ways, including using MATLAB plots (see subroutine POSTPR).

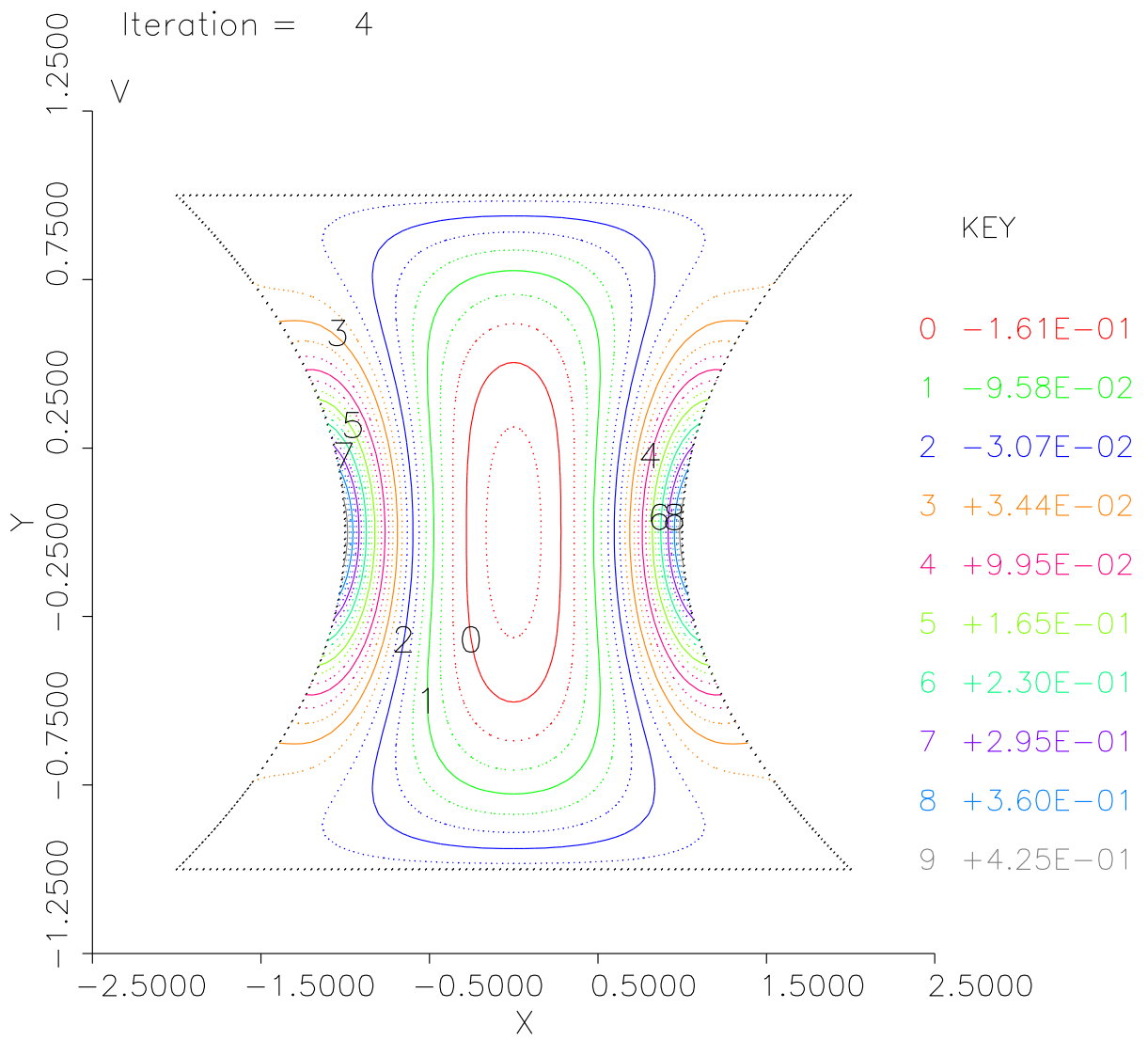
More information about PDE2D is contained in the book, "The Numerical Solution of Ordinary and Partial Differential Equations, second edition," Granville Sewell, John Wiley & Sons, 2005.

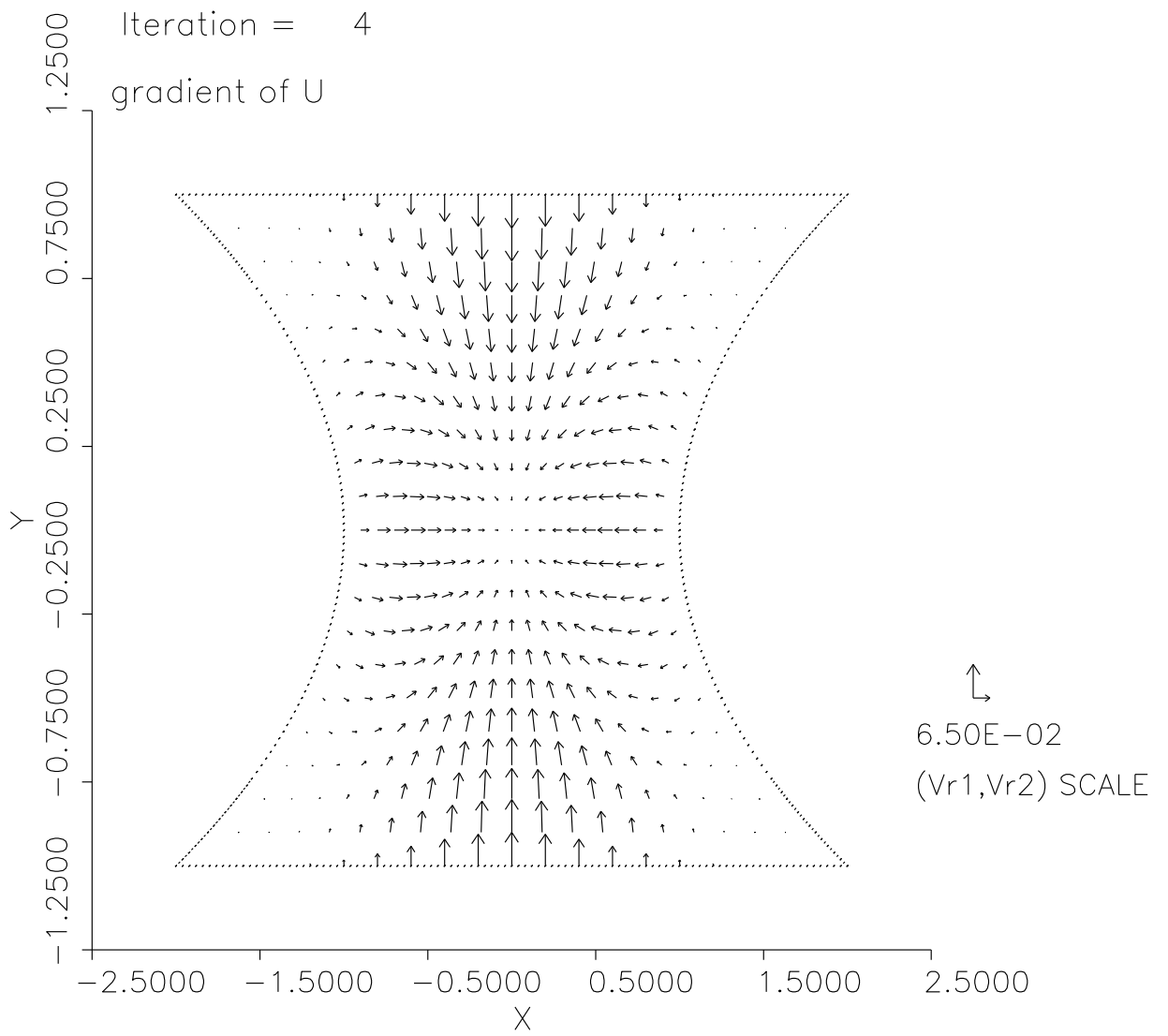


Integral output was 0.05826  
Grid and output graphs of U and V, and the gradient of U, are shown below:









MATLAB plot of U from \*.m file generated automatically by PDE2D

$T = 4$

