Find the conditional probabilities of the indicated events when two fair dice (one red and one green) are rolled.

a) The sum is 6, given that the green one is either 4 or 3.

Solution: Let A be the event that the sum of the two dice is 6, and let B be the event that the green one is either 4 or 3. We need to find $P(A \mid B) = \frac{P(A \cap B)}{P(B)}$. This means we need to compute the two quantities $P(A \cap B)$ and $P(B)$. To find $P(A \cap B)$, we must find $n(S)$ and $n(A \cap B)$, where $n(S)$ is the total number of possible outcomes when two dice are rolled. Since there are two dice and there are six possible outcomes for each die, $n(S) = 36$. Next, $n(A \cap B)$ is the total number of outcomes in which the sum is 6 **and** the green one is either 4 or 3. We can easily count the total number of ways this can happen: Since the green die must be 3 or 4, that means that the red die must be either 3 (if green is 3) or 2 (if green is 4), so the event space is $E = \{(4, 2), (3, 3)\}$, where the first entry of each ordered pair is the green one and the second entry is the red one. Thus, there are only two outcomes in which the sum is 6 **and** the green one is either 4 or 3, so $n(A \cap B) = 2$. This means $P(A \cap B) = \frac{2}{36} = \frac{1}{18}$.

To find $P(B)$, we must compute $n(B)$, the total number of outcomes for which the green die is either 4 or 3, then divide this by $n(S) = 36$. If the green die is a 4, the red die can be anything from 1 to 6, so there are 6 outcomes in which the red die is a 4. Similarly, there are 6 outcomes in which the green die is a 3. Thus, there are 12 outcomes in which the green die is either 4 or 3, and so $n(B) = 12$ and $P(B) = \frac{12}{36} = \frac{1}{3}$. Putting this all together, we find that

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{1/18}{1/3} = \frac{1}{18} \times \frac{3}{1} = \frac{3}{18} = \frac{1}{6}.$$

b) The red one is 4, given that the green one is 4.

Solution: Let A be the event that the red die is 4, and let B be the event that the green one is 4. We need to find $P(A \mid B) = \frac{P(A \cap B)}{P(B)}$. This means we need to compute the two quantities $P(A \cap B)$ and $P(B)$. To find $P(A \cap B)$, we must find $n(S)$ and $n(A \cap B)$. As in the last part, $n(S) = 36$. Next, $n(A \cap B)$ is the total number of outcomes in which both the red die and the green die are 4. There is only one way that this can happen (the event space is $E = \{(4,4)\}$), so $n(A \cap B) = 1$ and $P(A \cap B) = \frac{1}{36}$. Next, to find $P(B)$ we need to count the number of outcomes in which the green die is 4. There are 6 outcomes where this can happen, since the red die can be any number from 1 to 6 but the green die must be 4. Thus, $n(B) = 6$ and $P(B) = \frac{6}{36} = \frac{1}{6}$. Finally,

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{1/36}{1/6} = \frac{1}{36} \times \frac{6}{1} = \frac{6}{36} = \frac{1}{6}.$$