Suzy is given a bag containing 4 red marbles, 3 green ones, 2 white ones, and 1 purple one. She grabs five of them. Find the probabilities of the following events, expressing each as a fraction in lowest terms.

For all of these problems, $n(S)$ is the same. In this case, $n(S)$ is the number of ways you can choose five marbles from ten, so $n(S) = C(10, 5) = 252$. The goal here is to find $n(E)$ for each of the parts, then find the probability of each event using the formula $P(E) = \frac{n(E)}{n(S)}$.

a) She has all the red ones.

Solution: We need to find the number ways that Suzy can get all of the red marbles. Since there are four red ones and she's grabbing five marbles, the fifth marble must come from the non-red marbles. We can view this as a sequence of steps, where the first step in the sequence is to choose all the red ones. There is $C(4, 4) = 1$ way to do this. The next step is to choose the last marble from the six non-red marbles. There are $C(6, 1) = 6$ ways to do this. Since it is a sequence of steps, we use the multiplication principle. The total number of ways she can get all the red ones is $C(4, 4) \cdot C(6, 1) = 1 \cdot 6 = 6$. The probability of this event is $P(E) = \frac{6}{252} = \frac{1}{42}$.

b) She has at least one white one.

Solution: Whenever you see the words “at least” or “at most,” you are most likely going to have to consider more than one case. Here, “at least one white one” means that she can get either one white one or two white ones. We consider these cases separately and then add the results (this is the addition principle – these two events are disjoint since it is not possible for them to happen at the same time – you can never have both one white one and two white ones at the same time.) Case one is the case where she gets exactly one white marble. Since she is grabbing five marbles, this means that the other four must come from non-white marbles. This can be viewed as a sequence of steps, where the first step is to choose one white from two, then choose four marbles from the eight non-white marbles. There are $C(2,1) = 2$ ways to choose one white from two, and there are $C(8,4) = 70$ ways to choose four marbles from eight. Thus, there are $C(2,1) \cdot C(8,4) = 2 \cdot 70 = 140$ ways to get exactly one white marble. Case two is the case where she gets both white marbles. Using similar logic, there are $C(2,2) \cdot C(8,3) = 1 \cdot 56 = 56$ ways to get exactly two marbles (the sequence here is to choose two white marbles from two, then choose three marbles from eight). Adding the results from the two cases gives $n(E) = 140 + 56 = 196$. The probability of this occurring is $P(E) = \frac{196}{252} = \frac{7}{9}$.

c) She has two red ones and one of each of the other colors.

Solution: The sequence here is that we want to choose two red from four, then choose one green from three, then choose one white from two, then choose one purple from one. So, $n(E) = C(4,2) \cdot C(3,1) \cdot C(2,1) \cdot C(1,1) = 6 \cdot 3 \cdot 2 \cdot 1 = 36$. The probability of this occurring is $P(E) = \frac{36}{252} = \frac{1}{7}$.

d) She has at most one green one.

Solution: This part is similar to b), except that it's now "at most" instead of "at least." "At most one green one" means that she can have either one green one or no green ones. Again, we consider these two cases separately then add the results to find $n(E)$. For case one, there are $C(3,1) \cdot C(7,4) = 3 \cdot 35 = 105$ ways to get exactly one green one (first choose one green from three, then choose the remaining four marbles from the seven non-green ones). For case two, there are $C(7,5) = 21$ ways to get no green marbles (all five marbles must be chosen from the seven non-green ones). Adding gives $n(E) = 105 + 21 = 126$. The probability that you get at most one green marble is $P(E) = \frac{126}{252} = \frac{1}{2}$.